Renewable Technology Adoption Costs and Economic Growth*

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Abstract

We develop a dynamic general equilibrium integrated assessment model that incorporates costs due to new technology adoption in renewable energy as well as externalities associated with carbon emissions and renewable technology spillovers. We use world economy data to calibrate our model and investigate the effects of the technology adoption channel on renewable energy adoption and on the optimal energy transition. The calibrated model implies several interesting connections between technology adoption costs, the two externalities, and welfare. We investigate the relative effectiveness of two policy instruments-Pigouvian carbon taxes and policies that internalize spillover effects-in isolation as well as in tandem. Our findings suggest that renewable technology adoption costs are of quantitative importance for the energy transition. We find that the two policy instruments are better thought of as complements rather than substitutes.

JEL Codes: H21, O14, O33, Q54, Q55

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1 Introduction

We investigate the optimal transition from a primarily fossil-fueled world economy to a renewable-energy-fueled world economy. This transition depends on several components, including the relative costs and benefits of using different energy sources, the relative availability of fossil fuel, and the rate of technological progress in the energy sector. First, fossil fuel sources constitute an exhaustible resource. A second consideration involves climate factors. As fossil fuel use generates externalities through increasing the stock of greenhouse gas (GHG) emissions, the need for a clean substitute becomes increasingly apparent. In addition, technology spillovers imply the need to consider that investments in renewable technology create positive externalities. Our methodological contribution lies in exploring the quantitative significance of an additional, not well-studied factor in the process of new renewable technology adoption. In the presence of rapid technological progress and large capital costs, new technology adoption often involves certain additional costs. These can include costs associated with decommissioning, scrapping, or recycling old equipment; adjustment costs resulting from the switch; and legal and transaction costs associated with financing, selling, purchasing, and installing new equipment.[1] The greater the speed of improvements, the higher the costs resulting from early adoption. Our modeling of this effect applies more generally, but we believe it is particularly relevant for energy investments, as they tend to be capital intensive. In addition, renewable energy technologies are new and subject to relatively rapid technological progress as compared to fossil fuel. Our main contribution lies in exploring the quantitative significance of this channel in a dynamic general equilibrium integrated assessment model (IAM) that incorporates climate, as well as technological spillover externalities.

Although our aggregate modeling approach abstracts from individual sectors, the following example is instructive of the kind of effect we have in mind. Consider the market for electric vehicles (EV). Batteries are a significant fraction of the cost, often close to 30-40 percent of purchasing an EV. By any measure, including increased energy density and reduced cost, batteries have been steadily improving in recent years. Yet, in comparison, EV purchases have remained relatively flat over the same time horizon. It is important to note that for virtually all EVs, the battery technology at the time of purchase is "embedded" in the vehicle. That is, it is difficult or impossible to upgrade it, unless the vehicle is replaced with a new one. It is reasonable to expect that, concerned about characteristics such as overall range, some consumers might prefer to wait until sufficient progress justifies an EV purchase. This would be

[1] Mauritzen (2012) discusses scrapping patterns for less productive wind turbines in Denmark. We should emphasize that the purpose of this paper is not to provide theoretical micro-foundations for scrapping and related costs. Rather, we are investigating the quantitative implications of considering such costs for the energy transition. Costs associated with new technology adoption are, of course, also relevant and have been studied in the context of fossil fuel-derived energy and stranded assets, see Rezai and van der Ploeg (2019). To our knowledge, our paper is the first to study the implications of such costs for optimal renewable energy investment.

in line with our reasoning: there are costs associated with early adoption.[2]

A second focus of our study concerns the effectiveness of different climate policy instruments. While Pigouvian taxes on carbon emissions have several theoretical advantages and are generally favored by economists, they have proved difficult to implement in practice. As a partial substitute, many advocate policies that directly promote market penetration by renewables. We investigate the degree of substitutability between carbon taxes and policies that promote renewable energy by internalizing spillover externalities.

In our model, energy, capital, and labor are inputs in the production of final consumption goods. Energy can be produced from either fossil or renewable sources. Both require capital, which is also used in the production of the final good. At each point in time, productivity in the renewable energy production can increase as a result of adopting new capital. The actual improvement is subject to a spillover, as it depends on the aggregate investment in the renewable sector. We model the cost associated with replacing existing equipment as a temporary adverse productivity shock to the production function of renewable energy firms. These costs are assumed to be proportional to the size of the capital replacement in our analysis. Thus, our results are different from models of technological progress where the cost of adopting and utilizing new technologies is independent of the size of the existing stock of capital.

An important modeling choice concerns the degree of substitutability between different forms of energy. We will distinguish between substitutability in production versus in the consumption of energy. We assume that, once produced, renewable and fossil-fuel-derived energy are perfect substitutes in the production of the final good. A high substitutability seems a reasonable benchmark when considering long-run effects, as we do in this paper. For example, substitutability is justified in the presence of energy storage.[3] Importantly, fossil fuel and renewable forms of energy in our model are not perfect substitutes in terms of their required inputs, as they require different amounts of capital to produce. The evolving nature of renewable energy technologies will be a main focus, as we study properties of the optimal energy transition. While we do not consider technology advancements in fossil fuel in our benchmark case, we investigate the robustness of our findings to introducing technological progress in the fossil fuel sector.

We employ an IAM to characterize the optimal transition to a renewable-energy-fueled economy. The model is based on Golosov, Hassler, Krusell, and Tsyvinski (GHKT, 2014), who in turn build on Nordhaus's pioneering work in climate economics. We decentralize the optimum using a Pigouvian tax on GHG emissions together with a revenue-neutral policy on renewable technology

[2]For more on EVs see, for example, https://www.eia.gov/todayinenergy/detail.php?id=36312#, https://www.researchgate.net/figure/Evolution-of-battery-energy-density-and-cost-Global-EV-outlook-2016_fig3_309313559, and https://www.statista.com/statistics/797638/battery-share-of-large-electric-vehicle-cost/. EV purchases are often subsidized, which we ignored for this discussion.

[3]While not widely available currently, there are strong indications that such storage technologies will enter commercialization in the next decade or so.

adoption and find that the efficient energy mix involves a gradual decline in the use of fossil fuel. We then calibrate the model using world economy data.

The extent to which the world economy will use its available fossil fuel reserves depends on the assumption about the total endowment of accessible hydrocarbons. For realistic parametrizations, the transition is made well before exhaustion of these resources. In the absence of a Pigouvian tax on carbon emissions, policies that incentivize penetration by renewables through internalizing spillovers may provide relatively small benefits and can even be detrimental to short-run growth. In addition, we find that the decrease in global temperatures with respect to the status quo is negligible under this policy. In contrast, global temperatures decrease by about 10 percent if the optimal Pigouvian tax is in place. Similarly, the reduction in consumption of fossil fuel resulting from internalizing technology spillovers is significantly larger if the optimal Pigouvian tax is also in place. While the gains from internalizing the spillovers alone are small, comparing the status quo to the scenario where both policies are implemented results in a consumption-equivalent welfare gain of 1.431 percent. We conclude that when it comes to social welfare, carbon taxes and policies that promote renewable energy by eliminating spillover externalities are best thought of as *complements* rather than substitutes.

While theoretically desirable, Pigouvian taxes have generally proved difficult to implement in practice. For example, a short-lived government might choose a lower or no Pigouvian tax, as it is effectively more impatient than the representative agent. In the Appendix, we briefly discuss how the theoretical model can be extended to account for such considerations. The urgency of climate change has led to calls for action by several international organizations and governments around the globe. As a variety of mitigation measures are currently under consideration, our findings suggest that, in the presence of rapid technological progress and the associated costs, such as scrapping costs, some caution might be warranted before we conclude that direct subsidies are always a suitable substitute in the absence of a carbon tax.

Four parameters will be important for our calibration strategy: (1) the level of the spillover externality in renewable energy production, (2) the current "Pigouvian tax rate" (the ratio of the actual carbon tax relative to its optimal value), (3) the initial resources of fossil fuel, and (4) the productivity in the renewable energy technology. To calibrate these parameters, we will use observations from the world economy, together with theoretical relationships derived in the context of our model. For example, we will employ the fact that the change in the renewable energy share of the energy output decreases in the level of the spillover externality. Similarly, to get a handle on the current Pigouvian tax rate, we use the current level of fossil fuel consumption and the implication that an increase in the Pigouvian rate would result in a decrease in fossil fuel use. Importantly, the magnitude of the Pigouvian tax in our calibration should be interpreted as reflecting the difference between the opportunity cost of the average fossil fuel producer (which is rather low) and the average price paid by consumers of fossil fuel. Last, to calibrate the productivity in the renewable energy sector, we will use the current share of renewables together with the fact that their productivity is increasing in their share.

The calibrated model allows us to derive several interesting connections between new technology adoption, the two externalities, Pigouvian taxation, growth, and welfare. We investigate the quantitative importance and potential complementarity between the two policy instruments by studying their effects in isolation and in tandem. Our findings suggest that costs associated with new technology adoption are of quantitative importance along the energy transition. We find that when spillover externalities are internalized, renewable firms take on capital at a smaller scale, as they need to replace more of their capital in order to adopt newer technologies faster. This is due to the crucial assumption that the cost of technology adoption is proportional to the firm's capital stock. Our results also point to complementarities between carbon taxes and internalizing spillovers, with sizable welfare gains only in the case when both policies are present. This complementarity exists because the optimal Pigouvian tax reduces the initial negative effect from the optimal renewable adoption policy.

As a robustness check, we also calibrate the model taking into account technological developments in the fossil fuel sector. The magnitude of the Pigouvian tax remains virtually unchanged, but the value of the spillover externality is greater than in the benchmark case. We find that incorporating fossil fuel growth results in a faster transition to a fully renewable economy in the status quo case and a lesser welfare gain from switching to the optimal technology adoption, as the calibrated value is already closer to the optimal level. Importantly, the main mechanisms governing the interactions between Pigouvian taxation and renewable technology adoption, which are the focus of our paper, remain intact when technological progress in fossil fuel is incorporated.

1.1 Related Literature

Our paper contributes to the growing literature that uses IAM to study energy transitions, innovation, and growth. In the economic growth literature, Parente (1994) studied a model in which firms choose to adopt new technologies as they gain specific expertise through learning-by-doing. He identified conditions under which equilibria exhibit constant per capita output growth. As in most of the literature on innovation and growth, Parente abstracted from issues related to climate and energy, which are the focus of our study. Atkeson and Burstein (2015) study the impact of policy-induced changes in innovative investment and the implications for medium- and long-run innovation and growth. They, too, abstract from climate and energy considerations. Jensen and Traeger (2014) study optimal climate-related mitigation under uncertainty about technological progress and growth. Their analysis concentrates on long-term effects and explores different channels than ours. Throughout the paper we will abstract from the admittedly important strategic aspects resulting from the need to coordinate climate policy across different coalitions of countries, see, for example, Kollenbach and Schopf (2022).

Nordhaus (1994) pioneered the study of climate factors in dynamic economic modeling. Traditionally, a large part of the economic analysis of energy and environmental issues focuses on computable general equilibrium models that often abstract from endogenous technological progress.[4] Acemoglu et al. (2012) study a growth model that takes into consideration the environmental effects from operating "dirty" technologies. They consider policies that tax innovation and production in the dirty sectors. They find that subsidizing research in the "clean" sectors can speed up environmentally friendly innovation without the corresponding slowdown in economic growth.[5] Consequently, optimal behavior in their model implies an immediate increase in clean energy research and development (R&D), followed by a complete switch toward the exclusive use of clean inputs in production. We view our paper as complementary to theirs. We do not model directed technical change; instead, we introduce the replacement-cost channel associated with new capital adoption. While we think that their main policy recommendations are likely to remain valid, our quantitative findings suggest that the optimal rates of new technology adoption might be affected if we take such costs into account. More recently, Traeger (2021) considers a variety of modeling strategies and demonstrates the sensitivity of the optimal tax recommendation to different calibration assumptions.

As mentioned earlier, our analysis builds on GHKT (2014). They develop a tractable dynamic general equilibrium model that incorporates the feedback between energy use and the resulting climate consequences. They derive a formula and numerical values for the optimal tax on carbon emissions. They, however, abstract from the costs associated with endogenous technological progress. We will employ several elements from their work in what follows, including the tractable modeling of the environmental externality. Van der Ploeg and Rezai (2016,2021) extend the model in GHKT in several ways. They allow for general fossil fuel extraction costs, a negative impact of climate change on growth, mean reversion in climate damages, labor-augmenting and green technology progress, and a direct effect of the emissions stock on welfare. They discuss the social optimum as well as the optimal carbon tax, renewable energy subsidies, and the prospect of stranded assets.[6]

Acemoglu, Akcigit, Hanley, and Kerr (2016) also use the structure in GHKT to study questions related to the transition to clean technologies. They employ a "ladder" model to study technological progress in both the clean and the dirty sectors, and they estimate their model using R&D and patent data. They assume that increased representation of fossil fuel encourages further use, and that fossil fuel use stops after 200 years. They conclude that both Pigouvian

[4] See, for example, Nordhaus and Boyer (2000) and references therein.

[5] They assume a relatively high elasticity of substitution between dirty and clean energy and two types of externalities: (1) an environmental externality in the production of dirty energy, and (2) knowledge externalities related to R&D.

[6] Other related papers include Stokey (1998), who considers growth under environmental constraints; Goulder and Schneider (1999), who study endogenous innovations in abatement technologies; Van der Zwaan et al. (2002), who investigate the impact of environmental policies in a model with learning-by-doing; and Popp (2004), who considers innovation in the energy sector and the costs of environmental regulation. See also Hartley et al. (2016), who study technological progress and the optimal energy transition, and Van der Ploeg and Withagen (2011), who point to the possibility of a green paradox in this context.

taxation and renewable energy subsidies are needed in order to make the (optimal) transition sooner rather than later.[7] Renewable subsidies in their model encourage technological progress without overtaxing short-run future output. Fried (2018) develops a dynamic general equilibrium model to investigate the effect of carbon taxes in inducing innovation in green technologies. The carbon tax is chosen exogenously to match a 30 percent reduction in emissions in 20 years. The model introduces endogenous innovation in both dirty and green energy production as well as in the non-energy sectors and assumes positive spillovers between clean and dirty energy technologies. As a result, the paper finds that abstracting from endogenous innovation results in overestimating the size of the carbon tax needed to attain the given reduction in emissions.

More recently, Hassler et al. (2020) introduced a multi-region general equilibrium IAM. Their focus is on sub-optimal climate policies that may vary across regions. They consider the uncertainty associated with evaluating the cost from climate change and find that the costs of underestimating climate change are an order of magnitude larger than the costs of overestimating it. Thus, when various energy sources are sufficiently substitutable, ambitious climate policies can be thought of as effective insurance against adverse climate shocks. Their analysis shows that the economic costs of achieving climate goals increase substantially when some regions do not participate in mitigation. Finally, they find that when it comes to climate goals, green energy subsidies are a poor substitute for a Pigouvian tax on fossil fuel, particularly on coal. Their model does not explicitly study endogenous technical change.

More recently, Lemoine (2021) combines a calibrated model of directed technical change with the DICE integrated assessment model to study transition innovation dynamics. He finds that the economy gradually transitions from coal to gas and then to renewable energy even in the absence of related government policy. The optimal carbon tax in his model prescribes a rapid redirection of R&D and of energy supply towards renewables. Langer and Lemoine (2018) study dynamic considerations in designing subsidies to new technologies. Their setup involves a tradeoff between intertemporally price discriminating, by imposing a subsidy that increases over time, and taking advantage of future technological progress, by imposing a subsidy that decreases over time. They find that in the context of residential solar in California the subsidy should increase over-time.

We differ from this literature mainly in that we model the additional channnel associated with the costs of early technology adoption when technological progress is rapid. When technology is embedded in the current capital stock, subsidizing renewable resources encourages additional use of capital using the current technology. This, in turn, might not be optimal in the presence of replacement costs. Gowrisankaran and Rysman (2012) explore a similar effect in a different

[7]The reason is that energy is produced with the leading-edge energy technology. If clean technology is far behind, most research directed towards that sector will generate incremental innovations that cannot be competitive in the absence of high levels of carbon taxes. If green R&D subsidies can be maintained for a while, clean energy production gradually becomes self-sustained, as the clean technologies that can compete with the dirty ones expand due to a series of incremental innovations.

context. They study optimal decisions by consumers choosing the timing of purchase among an expanding set of available camcorders. While consumers usually own only one camcorder at a time, they may substitute an old camcorder with a new one. As prices, quality, and variety improve over time, waiting is valuable in their model. They argue that initial market share for digital camcorders was modest, as forward-looking consumers were rationally expecting that cheaper and better players would appear in the future. A related effect is explored in Manuelli and Seshadri (2014). They focus on technology diffusion of tractors in American agriculture during the first part of the 20th century. They argue that part of the reason for the slow rate of adoption was that tractor quality kept improving over that period. As a result, farmers chose to postpone their purchase, rather than investing in a tractor that would soon become obsolete. The main contribution of our paper is to explore this channel in the context of renewable energy technology adoption.

The paper proceeds as follows. Section 2 introduces the model. Sections 3 and 4 discuss efficiency, equilibrium, and optimal policy. Section 5 outlines our calibration strategy. Section 6 presents our main quantitative findings, including those from an extension of the base model that accounts for productivity improvements in the fossil fuel sector. A brief conclusion follows. The Appendix contains technical material and some extensions.

2 The Base Model

Our model incorporates a version of the neoclassical growth model, together with energy, technology, and climate factors. Time is discrete and the horizon is infinite, $t = 0, 1, \ldots$. There is a single consumption good per period, and all markets are competitive. The economy is populated by a representative infinite-lived household that discounts the future at rate $\beta \in (0,1)$ and values period t consumption, c_t, through a utility function $u(c_t)$. We assume that u is smooth, strictly increasing, and strictly concave, and that the usual Inada conditions hold. The labor endowment is normalized to 1, and labor is supplied inelastically. There are three different types of firms, all owned by the household: the final-good firm as well as two types of intermediate-good firms that produce energy from fossil fuel and from renewable sources, respectively. In each period, the household chooses how much capital, k_t, to rent at rate r_t and receives profits resulting from the firms' activities. All capital depreciates at rate $\delta \in (0,1)$.

The representative final-good-producing firm produces output, y_t, using capital, k_t^c, labor, l_t, and energy, e_t. Production can be affected by "environmental quality," indexed by Γ_t, which reflects the total stock of GHGs in the atmosphere. Ignoring environmental damages, the final good production function is given by

$$y_t \leq F\left(k_t^c, l_t, e_t, \Gamma_t\right). \tag{1}$$

We assume that Γ_t, affects output through a damage function $D_t(\Gamma_t)$, and that damages are multiplicative. Thus, the final good production function becomes

$$F\left(k_t^c, l_t, e_t, \Gamma_t\right) = (1 - D_t(\Gamma_t))\widetilde{F}\left(k_t^c, l_t, e_t\right), \tag{2}$$

where $1 - D_t(\Gamma_t) = \exp\left[-\pi_t\left(\Gamma_t - \overline{\Gamma}\right)\right]$. Here, $\overline{\Gamma}$ represents the pre-industrial GHG concentration in the atmosphere, and π_t is a variable that parametrizes the effect of higher GHG concentrations on damages. The function D captures the mapping from the stock of GHG, Γ_t, to economic damages measured as a percentage of output. We assume that $\widetilde{F}\left(k_t^c, l_t, e_t\right)$ has a Cobb-Douglas form:

$$\widetilde{F}\left(k_t^c, l_t, e_t\right) = \widetilde{A}_t(k_t^c)^{\theta_k}(l_t)^{\theta_l}(e_t)^{1-\theta}, \tag{3}$$

where $\widetilde{A}$ is a productivity parameter, while θ, θ_k, $\theta_l \in (0, 1)$, and $\theta_k + \theta_l = \theta$. Thus, the final good production function can be rewritten as

$$y_t \leq A_t(k_t^c)^{\theta_k}(l_t)^{\theta_l}(e_t)^{1-\theta}, \tag{4}$$

where $A_t \equiv (1 - D_t(\Gamma))\widetilde{A}_t$. The level of GHG evolves according to

$$\Gamma_t - \overline{\Gamma} = \sum_{n=0}^{t-T}(1 - d_n)f_{t-n}, \ t \geq T, \tag{5}$$

where $d_n \in [0, 1]$, and f_{t-n} indicates the anthropogenic GHG emissions in period $t - n$. The variable $1 - d_n$ represents the amount of carbon that remains in the atmosphere n periods into the future, and T defines the start of industrialization.

The depreciation structure in (5) is characterized by three parameters. It is assumed that a fraction φ_L of emitted carbon stays in the atmosphere forever, while a fraction $(1-\varphi_0)$ of the remaining emissions exit into the biosphere. The remaining part decays at geometric rate φ. Thus,

$$1 - d_n = \varphi_L + (1 - \varphi_L)\varphi_0(1 - \varphi)^n. \tag{6}$$

The level of the GHG concentration can be then decomposed to a permanent part, Γ_t^p, and a decaying part, Γ_t^d:

$$\Gamma_t = \Gamma_t^p + \Gamma_t^d, \tag{7}$$

where

$$\Gamma_t^p = \Gamma_{t-1}^p + \varphi_L f_t, \tag{8}$$

$$\Gamma_t^d = (1 - \varphi)\Gamma_{t-1}^d + (1 - \varphi_L)\varphi_0 f_t. \tag{9}$$

Energy can be produced by using fossil or renewable sources. We distinguish between substitutability in production versus substitutability in consumption between the two forms of energy. More precisely, we assume that energy derived from fossil fuel and that derived from renewable sources are perfect substitutes in the production of the final good.[8] As we measure fossil fuel use in units of

[8] Hassler et al. (2020) allow for imperfect output substitutability in energy and concentrate on medium-run questions.

carbon content, the flow of anthropogenic GHG emissions equals f_t, the fossil fuel used in energy production in period t. Let ϖ_t denote the available stock of fossil fuel in period t. Given ϖ_0, the law of motion for ϖ_t is

$$\varpi_{t+1} \leq \varpi_t - f_t. \tag{10}$$

We assume imperfect substitutability when capital is used in the production of different forms of energy. Fossil-fuel-derived energy production uses fossil fuel and capital as inputs according to production function

$$e_t^f \leq A_f (f_t)^{1-\alpha_f} \left(k_t^f\right)^{\alpha_f}, \tag{11}$$

where $A_f > 0$ and $\alpha_f \in (0, 1)$. This specification captures that, by using additional capital, more energy can be extracted from the remaining fossil fuel reserves. Thus, using more capital (higher k_t^f), or additional technological progress (higher A_f), in the fossil fuel sector lead to a reduction in emission intensity through improvements in fuel efficiency.[9]

We assume a measure one of competitive renewable-energy-producing firms. Constant returns to scale in capital and in productivity imply that the number of firms does not affect the aggregate industry variables.[10] Thus, our modeling abstracts from issues related to free entry, which is not a focus for our analysis. As firms in our model are heterogenous, we need to keep track of the identity, j, of each individual firm. The renewable energy output of firm j is given by

$$e_{j,t}^r \leq \Psi(i_{j,t}) \left(\mathcal{E}_{j,t}\right)^{1-\alpha_r} \left(k_{j,t}^r\right)^{\alpha_r}, \tag{12}$$

where $\mathcal{E}_{j,t}$ is firm j's productivity parameter, $\mathcal{E}_{j,0}$ is given for all j, and $\alpha_r \in (0, 1)$. We interpret $i_{j,t}$ as the *new technology adoption rate* by firm j in period t.

New technology adoption boosts future productivity, but implies a cost in terms of a contemporaneous output loss. This cost is increasing in the adoption rate and, moreover, it is proportional to the renewable firm's current production. If a renewable firm does not adopt a newer technology, then it can fully utilize its current capital stock under the current embedded technology without any disruption. However, if it chooses to improve the technology embedded in next period's capital, then during the current period it needs to replace its current capital with capital that uses an improved technology. Thus, the productivity improvement will result in a temporary disruption to the firm's current production.[11] It is natural to assume that the output loss will be proportional to the

[9] Needless to say, there are physical limits to how much energy can be extracted from a unit of fossil fuel. Our specification captures the increasing, yet concave, role of capital in this extraction prior to reaching those physical limits.

[10] We assume that if two firms merge, their resulting combined productivity will be in proportion to their capital stocks.

[11] One could think of this as a "time-to-build" constraint in adopting newer technologies, which is akin, yet distinct, from the standard "time-to-build" constraint for physical capital.

firm's current production.[12]

More precisely, we assume that the new technology adoption rate, i, reduces firm j's current output by a factor $\Psi(i_{j,t}) \geq 0$, where $\Psi(\cdot)$ is such that $\Psi(0) = 1$, $\Psi'(\cdot) < 0$, $\Psi''(\cdot) < 0$, and $\Psi(\bar{i}) = 0$, for some $\bar{i}$. We interpret this as the total cost associated with the replacement of current equipment.[13] This implies that capital replacement costs are independent of the depreciation rate.

We will consider the possibility of a spillover effect, where the productivity of each individual firm also depends on aggregate technology adoption. Put differently, as more firms adopt new technologies, the benefits affect the entire renewable energy sector. This creates an externality, leading to a discrepancy between equilibrium and optimal levels of new capital adoption.[14] We consider this effect to be especially relevant, as investments in the energy sector tend to be capital intensive. Thus, if innovators do not expect to capture the resulting returns, under-adoption of new technologies relative to the optimum is likely to occur.[15] More precisely, the productivity of firm j evolves according to

$$\ln \mathcal{E}_{j,t+1} \leq \xi i_{j,t} + (1-\xi)\left(\int_0^1 i_{j,t} k_{j,t}^r dj \Big/ \int_0^1 k_{j,t}^r dj\right) + \ln \mathcal{E}_{j,t}, \tag{13}$$

where $0 \leq \xi \leq 1$ parametrizes the strength of the spillover effect. The case where $\xi = 1$ corresponds to no spillovers, while $\xi = 0$ corresponds to the other extreme, where productivity is entirely determined by spillovers. In order to abstract from any size-dependent advantage to firms, the above expression normalizes each firm's technology adoption by its capital stock.

The production factors are allocated freely across sectors. Total capital used

[12] Alternatively we could introduce different "vintages" of renewable capital and stochastic depreciation, so that older and newer vintages coexist in production. To improve overall productivity, a firm would then need to replace older vintages before their full depreciation with more productive new ones. Thus, older vintages would be removed prior to their natural depreciation. This alternative way of modeling would naturally lead to a distribution of vintages within and across firms. To avoid the resulting complexity, and since distributional issues are not the focus of our study, we adopted a "reduced-form" approach to modeling this effect. There is extensive literature on dynamic vintage-capital-related models. See, for example, Benhabib and Rustichini (1991), Chari and Hopenhayn (1991), Greenwood, Hercowitz, and Krusell (1997), and Jovanovic (2012). Boucekkine, De La Croix, and Licandro (2017) provide a recent review.

[13] Admittedly, important innovation also takes place in the fossil fuel sector. Mainly for simplicity, we will concentrate on technological progress in the renewable sector in our baseline model. We will later extend our model to account for technological progress in fossil fuel. This leads to some quantitative differences, but the qualitative features of our results remain intact.

[14] The infant industry argument is sometimes used to justify subsidising the production of renewable energy. According to this argument, since fossil fuel technologies are more mature, renewable technologies cannot compete on an equal basis since scale and benefits from learning-by-doing can only be achieved under a larger market share.

[15] Bosettia et al. (2008) argue that international knowledge spillovers tend to increase the incentive to free-ride, thus decreasing investments in energy R&D. Braun et al. (2009) perform an empirical study of spillovers in renewable energy. They document significant domestic and international knowledge spillovers in solar technology innovation as well as significant international spillovers in wind.

in the economy cannot exceed the total supply; i.e., for all t,

$$k_t^c + k_t^f + \int_0^1 k_{j,t}^r dj \leq k_t. \tag{14}$$

In addition, the energy used in the production of the final good cannot exceed the total supply of energy:

$$e_t \leq e_t^f + \int_0^1 e_{j,t}^r dj. \tag{15}$$

The next section discusses optimal allocations for our model economy.

3 Efficiency

We begin by characterizing allocation efficiency in terms of some key relationships. We will later compare efficient outcomes to market allocations. The social planner chooses a sequence $\{c_t, k_t^c, k_t^f, f_t, e_t^f, \Gamma_t^p, \Gamma_t^d, \{i_{j,t+1}, k_{j,t}^r, \mathcal{E}_{j,t+1}, e_{j,t}^r\}_j\}_{t=0}^{\infty}$, $j \in [0,1]$, to solve the following problem:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t),$$

subject to (8)-(15) and

$$c_t + k_{t+1} \leq (1 - D_t(\Gamma_t^p + \Gamma_t^d)) \left[\widetilde{A}_t (k_t^c)^{\theta_k} (l_t)^{\theta_l} (e_t)^{1-\theta} \right] + (1-\delta) k_t, \tag{16}$$

as well as non-negativity constraints and given the initial values for the stock variables.

We let μ_F denote the Lagrange multiplier on the production constraint in the fossil fuel sector (equation (11)) and μ_r^j be the multiplier on the production constraint for renewable energy firm j (equation (12)). Similarly, we let $\mu_{\mathcal{E}}^j$ be the multiplier for the evolution of firm j's productivity (equation (13)). Finally, μ_K and μ_E are the multipliers associated with the distribution of the capital stock across sectors and with the supply of energy (equations (14) and (15)), respectively.

The first-order condition (FOC) with respect to e_{jt}^r gives

$$\int_0^1 \mu_{r,t}^j dj = \mu_{E,t}. \tag{17}$$

Moreover, the marginal utility from producing an extra infinitesimal amount of renewable energy should be equal across firms; i.e.,

$$\mu_{r,t}^j = \mu_{r,t}^h, \text{ for any two firms } j \text{ and } h. \tag{18}$$

The FOC with respect to $k_{j,t}^r$ gives

$$(1-\xi)\left(\frac{i_{j,t}-\overline{i_t}}{\overline{k_t^r}}\right)\int_0^1 \mu_{\mathcal{E},t}^j dj+\Psi\left(i_{j,t}\right)\alpha_r\mu_{r,t}^j\left(\frac{\mathcal{E}_{j,t}}{k_{j,t}^r}\right)^{1-\alpha_r}=\mu_{K,t}, \qquad (19)$$

where $\overline{k_t^r}=\int_0^1 k_{j,t}^r dj$, and $\overline{i_t}=\int_0^1 i_{j,t}k_{j,t}^r dj/\int_0^1 k_{j,t}^r dj$. Since (17) and (18) imply that $\mu_{r,t}^j=\mu_{E,t}$, equation (19) implies that the only non-aggregate variable that influences $i_{j,t}$ is $\frac{\mathcal{E}_{j,t}}{k_{j,t}^r}$.

The FOC with respect to $i_{j,t}$ gives

$$-\mu_{r,t}^j\Psi'(i_{j,t})\left(\frac{\mathcal{E}_{j,t}}{k_{j,t}^r}\right)^{1-\alpha_r}=\xi\frac{\mu_{\mathcal{E},t}^j}{k_{j,t}^r}+(1-\xi)\frac{\int_0^1\mu_{\mathcal{E},t}^j dj}{\overline{k_t^r}}. \qquad (20)$$

Finally, the FOC with respect to e_t^f gives

$$\mu_{E,t}=\mu_{F,t}. \qquad (21)$$

Since $\mu_{r,t}^j=\mu_{E,t}=\mu_{F,t}$, and $i_{j,t}$ is a function of $\frac{\mathcal{E}_{j,t}}{k_{j,t}^r}$, we have that $\frac{\mu_{\mathcal{E},t}^j}{k_{j,t}^r}$ is also a function of $\frac{\mathcal{E}_{j,t}}{k_{j,t}^r}$. The following result greatly simplifies our analysis. It asserts that if $\mathcal{E}_{j,t}$ and $k_{j,t}^r$ are proportional to the initial values of $\mathcal{E}_{j,0}$, then $i_{j,t}=i_t$, for all j and t. In other words, although renewable-energy-producing firms are heterogeneous, efficiency implies that they choose identical levels of i_t.

Proposition 1 *In an efficient allocation,* $\frac{k_{j,t}^r}{\mathcal{E}_{j,t}}=\frac{k_t^r}{\mathcal{E}_t}$ *and* $i_{j,t}=i_t$*, for all* j*.*

Proof. For any initial values of $\mathcal{E}_{j,0}$, there is a solution such that $\mathcal{E}_{j,t}$, $k_{j,t}^r$, $\mu_{\mathcal{E},t}^j$, and $\mu_{r,t}^j$ are proportional to the initial values of $\mathcal{E}_{j,0}$. Then (20) implies that $i_{j,t}=i_t$, for all $j\in[0,1]$. From (19), $\frac{\mathcal{E}_{j,t}}{k_{j,t}^r}$ is a function of $i_{j,t}$ only. As $i_{j,t}=i_t$, we have $\frac{\mathcal{E}_{j,t}}{k_{j,t}^r}=\frac{\mathcal{E}_t}{k_t^r}$. ■

4 Equilibrium and Optimal Policy

We derive the competitive equilibrium FOC for consumers and firms in the Appendix. Using these, we first characterize the equilibrium choice of investment in the renewable technology. In what follows, we let $\Phi_t\left(i_{j,t}\right)$ stand for the government policy conditional on a renewable firm's investment. Provided that $\xi<1$, in the absence of government policy this investment will be lower than optimal. Of course, the magnitude of the distortion depends on the level of the externality, ξ.

Proposition 2 *In a competitive equilibrium with* $\Phi_t\left(i_{j,t}\right)=0$*,* $i_{j,t}$ *is lower than optimal when* $\xi<1$*.*

The proof is given in the Appendix. Optimal policy needs to take into account two distortions. First, there is under-investment in i_t due to the spillover effects. The second distortion is due to the social costs associated with the climate externality. The next Proposition demonstrates that both distortions can be fully accommodated through the use of two instruments. First, a policy that taxes firms in proportion to their under-investment in i_t restores optimal investment by making firms indifferent between paying a "penalty" or pursuing the optimal level of investment. Second, a Pigouvian tax internalizes the externality from carbon emissions. As in GHKT (2014), under the special assumptions of log utility and 100 percent depreciation of capital, the Pigouvian tax imposed on the fossil fuel firms grows at the growth rate of the economy.

Proposition 3 *(1) The optimal allocation can be supported by a combination of a revenue-neutral policy,* $\Phi_t^j\left(i_{j,t}\right) = -(1-\xi)p_t^e\Psi'(i_t^*)\left(\frac{e_{j,t}^{*r}}{\Psi(i_{j,t}^*)}\right)(i_{j,t}-i_t^*)$*, imposed on renewable firms, together with a Pigouvian tax on fossil fuel use,* $\tau_t^f = \sum_{j=0}^{\infty}\beta^j\frac{u'(c_{t+j}^*)}{u'(c_t^*)}\pi_{t+j}y_{t+j}^*(1-d_j)$*, where* p_t^e *is the price of energy,* $\{c_t^*, y_t^*, i_t^*\}_{t=0}^{\infty}$ *is the solution to the planner's problem, and* $1-d_j = \varphi_L + (1-\varphi_L)\varphi_0(1-\varphi)^j$*. (2) If* $u(c) = \log(c)$*,* $\alpha_r = \alpha_f = \alpha$*,* $\pi_t = \pi$*, all* t*, and* $\delta = 1$*,* $\tau_t^f = y_t\pi\left[\frac{\varphi_L}{1-\beta} + \frac{(1-\varphi_L)\varphi_0}{1-(1-\varphi)\beta}\right]$ *grows at the growth rate of the economy.*

The proof is given in the Appendix. The optimal policy in our model has several interesting implications for our quantitative investigations, which are the focus of what follows. First, the policy on renewable energy firms generates no revenue, but it reduces the household's current profits from the renewable sector, as a result of inducing additional innovation compared to the competitive equilibrium. Second, the Pigouvian tax reduces the household's profits from the fossil fuel sector. However, the household receives a lump-sum transfer of equal magnitude; thus, its budget constraint remains unchanged. Finally, there is a separation between the two schemes, as the total effect on the household's budget is the same as the resource cost of innovation in the planner's problem.

For the remainder of the paper we will assume that $u(c) = \log(c)$ and will set $\delta = 1$. Moreover, we will assume that the stock of fossil fuel is large enough so that consumption of fossil fuel is not constrained. This assumption also allows us to abstract from dealing with fossil fuel ownership in the decentralized problem. In the Appendix we solve the constrained planner's problem backward, from a final state, where only renewable energy is used, and we confirm that the consumption of fossil fuel is endogenously bounded. In other words, the full transition to the renewable energy regime takes place prior to the exhaustion of fossil fuel. This is due to the growing productivity in the renewable energy sector eventually surpassing a threshold that makes fossil fuel the less efficient source. While allocating additional capital to the fossil fuel sector increases the production of energy per unit of fossil fuel, the present value of the marginal environmental damages limits the overall benefit from fossil fuel use. In the next section we calibrate our model in order to study the optimal timing of the transition to a renewable energy regime, as well as the effects of the GHG accumulation prior to this transition. This will allow us to explore the quantitative significance of the technology adoption effect on optimal policy and welfare.

5 Calibration

In this section we calibrate our model in order to study the transition from the current, predominantly fossil fuel economy, to an economy that fully relies on renewable energy. We use the calibrated model to evaluate the interaction between the two policy instruments: (1) the Pigouvian taxation on carbon emissions, and (2) the technology adoption targeting for renewable energy firms. More precisely, we evaluate how the two policies would affect the share of renewable energy, the accumulation of GHG, global temperatures, economic growth, and welfare, first in isolation and then in tandem. This will allow us to quantify the significance of the technology adoption effect and to explore the potential substitutability between the two policy tools.

The model's parameters can be divided into four categories related to preferences, technology, environmental damages, and the current (status quo) policies in place. We will use a log utility function and a benchmark annual discount rate of 4 percent, which gives $\beta = 0.96^{10}$, as a period is calibrated to 10 years.[16] Given the length of the period, there is some justification in considering the benchmark case of full depreciation of capital, $\delta = 1$. Turning to the aggregate Cobb-Douglas production function, we set the share of capital and labor, respectively, to $\theta_k = (1/3) \times 0.95$ and $\theta_l = (2/3) \times 0.95$, which imply an energy share of $1 - \theta = 1 - (\theta_k + \theta_l) = 0.05$.[17] We set the productivity growth rate in the final good sector so that the balanced growth rate is 2 percent, while the long-run population growth rate is set to zero. The production functions in the renewable energy sector and fossil fuel energy sector are constant returns to scale Cobb-Douglas functions with capital share equal to 0.5. We wish to clarify that the effect of costs associated with technology adoption, such as scrapping costs, remains intact even under 100 percent capital depreciation. The best analogy would be in the context of a continuous-time model. Although capital depreciates fully after one period (10 years), in a continuous time model, technology adoption costs, such as scrapping costs, would affect the productivity of renewable energy in the interim. As we work in discrete time, the costs imposed in the beginning of a period in our setup can be interpreted as capturing the average of such costs over the ten-year period when this capital is used. We also discuss a continuous time representation of these costs in the Appendix.

We assume the following form for the renewable technology adoption cost function, Ψ:

[16] Most macroeconomic studies use yearly discount rates between 2 percent and 5 percent.
[17] These are the values of the parameters for the model in Hassler et al. (2021).

$$\Psi(i) = \left(1 - \left(\frac{i}{\bar{i}}\right)^{\psi}\right)^{1/\psi}.$$

This functional form satisfies the earlier assumptions that $\Psi(0) = 1$, $\Psi'(\cdot) < 0$, $\Psi''(\cdot) < 0$, and $\Psi(i) = 0$, for $i = \bar{i}$. Moreover, the elasticity of the technology adoption cost with respect to the adoption rate is given by

$$-\frac{\Psi'(i)}{\Psi(i)} = \frac{1}{i} \times \frac{(i/\bar{\imath})^{\psi}}{1 - (i/\bar{\imath})^{\psi}}. \tag{22}$$

As shown in the Appendix, this elasticity plays an important role in determining both the long-run and the transitional technology adoption rate in the renewable sector. The parameter ψ provides us with a degree of freedom to match a long-run adoption rate that is consistent with the long-run growth rate of the economy. To calibrate Ψ, we need to assign values to two parameters: $\bar{i}$ (the highest possible technology adoption rate in the renewable sector) and ψ. We will use the relationship between $\Psi(\cdot)$ and the optimal asymptotic long-run growth rate of i^l, the productivity in the renewable sector. As we show in the Appendix, if the spillover externality is fully internalized, the long-run i^l is given by

$$-\frac{\Psi'(i^l)}{\Psi(i^l)} \quad = \quad \frac{\beta}{1-\beta}(1-\alpha).$$

Combining the above equation with (22) gives

$$\bar{i} = \left(1 + \frac{1}{\left(\frac{\beta}{(1-\beta)}(1-\alpha)i^l\right)}\right)^{1/\psi} \times i^l. \tag{23}$$

Thus, for a given i^l, a lower ψ implies a higher $\bar{i}$. In our baseline calibration, we set $\bar{i}$ to its maximum attainable level, which corresponds to setting ψ to its lowest possible level; i.e., by setting $\psi = 1/(1-\alpha) = 2$.[18]

To determine i^l, note that an asymptotically balanced growth path requires equal asymptotic growth rates between the renewable energy sector (which is the only source of energy in the long run) and the final good sector. This, in turn, implies $i^l = \log\left(g^l \times (g^c)^{\frac{1}{\theta_l}}\right) = 0.198$. Using (23), we set $\bar{i} = 0.489$.[19]

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[18]If $\psi = 1/(1-\alpha) < 2$, then $\Psi(i)^{\frac{1}{a-\alpha}}$ is no longer a concave function and the cost of technological adoption is no longer increasing.

[19]For sensitivity, we also set $\bar{i}$ to levels corresponding to 10 percent and 20 percent slower than the maximum attainable renewable technology growth rate. The slower rates correspond to setting $\bar{i}$ to 0.449 and 0.408, respectively. These translate to a maximum attainable annual growth rates of 4.6 percent and 4.2 percent, respectively, instead of the 5.0 percent baseline. In our robustness exercise we will accordingly set ψ equal to 2.21 and 2.50, respectively, in order for $\bar{i}$ to satisfy (23).

We follow GHKT (2014) in our calibration of the environmental damage parameters and the computation of the Pigouvian carbon tax. In particular, we set $\pi = 2.379 \times 10^{-5} \times 10$, $\varphi = 0.0228$, $\varphi_L = 0.2$, and $\varphi_0 = 0.393$. The optimal carbon tax follows from the last part of (83) in the Appendix and is given by

$$\mathcal{T}_P/y = \pi \left[\frac{\varphi_L}{(1-\beta)} + \frac{(1-\varphi_L)\varphi_0}{(1-\beta(1-\varphi))} \right]. \tag{24}$$

Given our calibration, this equation implies that $\mathcal{T}_P/y = 3.55 \times 10^{-4}$, which is equivalent to a tax of \$24.9 per ton. This is broadly consistent with the climate economics literature given the assumed level of discounting.[20]

We chose 2015 as our base year. Four parameters related to the energy sector remain to be calibrated: (1) the current stock of fossil fuel, W_0; (2) the current productivity of the renewable sector, $\mathcal{E}_0$; (3) the current Pigouvian tax level, τ^f; and (4) the spillover from the renewable technology adoption, ξ. For our baseline calibration we set $W_0 = 666GtC$.[21]

We set $\mathcal{E}_0$, τ^f, and ξ to match three data moments: (1) the current share of renewable energy in total energy production, s_0, (2) the current consumption of fossil fuel, f_0; and (3) the change in the share of renewable energy in the last period (10 years), $s_0 - s_{-1}$. The current productivity of the renewable sector affects the renewable share in total energy production. In turn, the spillovers from the renewable technology adoption affect the change in the productivity of the renewable sector and, thus, the change in the share of renewable energy. In addition, the Pigouvian tax affects the use of fossil fuel and, consequently, the share of fossil fuel and renewable sources in total energy production. In what follows, we denote by τ^f the value of the Pigouvian tax as a percentage of its optimal level, τ^*. Setting $\mathcal{E}_0 = 14.92$, $\tau^f = 0.63 \cdot \tau^*$, and $\xi = 0.54$, our model matches $f_0 = 100\ GtC$,[22] $s_0 = 10.2$ percent, and $s_0 - s_{-1} = 2.3$ percent.[23]

6 Quantitative Findings

Our calibration allows us to evaluate the quantitative significance of the technology adoption effect as well as the effects of the carbon tax and the renewable

[20] See Figure 2 in GHKT (2014).

[21] See Section 4.3 in Li, Narajabad, and Temzelides (2016).

[22] See EPA: https://www.epa.gov/ghgemissions/global-greenhouse-gas-emissions-data#Trends.

[23] This includes all modern plus traditional renewables (including biomass). We calculated an initial 10-year growth rate of 4.7 percent for renewables, with a corresponding rate of 2 percent for the entire energy sector (we excluded nuclear energy from this calculation). See https://www.ren21.net/reports/global-status-report/. We remark that while, with a few exceptions, an explicit carbon tax is largely absent in most countries, several uses of fossil fuel are taxed at relatively high rates. Gasoline and other fuel related to transportation are a leading example. The initial value of τ^f should be interpreted in that light, as the difference between the marginal cost to fossil fuel producers and the average price paid by consumers of fossil fuel. Of course, the market for fossil fuel shares many characteristics of an oligopoly. Taxes and subsidies of fossil fuel and related energy sources are disregarded in the GHKT (2014) calibration.

adoption policy in isolation and in tandem. We simulate our model considering different scenarios for the two policy parameters. Figure 1 below shows the paths for the share of renewable energy (top), accumulated fossil fuel consumption (middle), and global temperatures (bottom) in each respective policy scenario. The dotted, dashed, dot-dashed, and solid lines indicate the status quo benchmark (business as usual), optimal technology adoption, optimal Pigouvian tax, and combined optimal policies (full optimum), respectively. Clearly, the outcomes under either an optimal Pigouvian tax policy alone or the optimal technology adoption policy alone differ significantly from the outcome when both policies are present. This thought experiment helps us understand how the two policies interact in the presence of the technology adoption channel. Next, we comment on each panel individually.

The first panel gives the share of renewables in energy production as a function of time under the different policy scenarios. Note that the optimal technology adoption policy leads to a "rotation" of the status quo path, while the Pigouvian tax "shifts" the status quo path along the transition. As a result, setting technology adoption to its optimal level in the absence of the optimal Pigouvian tax *reduces* the share of renewables in the short run relative to the status quo. At the same time, the full switch to renewable energy production occurs somewhat earlier than in the benchmark case. In contrast, setting the Pigouvian tax to its optimal level in the absence of a policy inducing optimal technology adoption increases the share of renewable energy immediately. In the full optimum, setting both policies to their combined optimal levels reduces the short-run share of renewable energy. However, the transition to a fully renewable global economy takes place by 2070, the earliest among the four scenarios.[24]

The second panel describes the evolution of cumulative fossil fuel consumption in the same four scenarios. Interestingly, absent a tax on GHG emissions, the cumulative fossil fuel consumption is initially somewhat more intense if the technology externality is internalized than in the status quo. This is because the faster growth in renewable energy productivity allows the economy to rely fully on renewable energy earlier. Similarly, when both the Pigouvian tax and the technology adoption are set to their optimal levels, the economy reaches the fully renewable energy state earlier and more fossil fuel is left unused. Consistent with the "Green paradox," this also implies a heavier use of fossil fuel initially than in the case where the Pigouvian tax is in place but the renewable policy is absent.[25] By comparison, GHKT (2014) find that, while increasing in the laissez-faire (zero-tax) case, optimal consumption of fossil fuel stays relatively

[24] Note that the fossil fuel consumption drops to zero in finite time, not just asymptotically. The reason for this is that fossil fuel and renewable energy are assumed to be perfect substitutes in consumption. Thus, as the consumption of fossil fuel vanishes, its marginal productivity, which depends on the marginal productivity of energy, remains finite. Since damages from emissions grow proportionally to GDP, there is a point after which the productivity of renewables becomes high enough to make fossil fuel obsolete.

[25] Often, a Green paradox attributes the rise of current consumption of fossil fuel to expectations about future policies that will make consumption of fossil in the future less advantageous. In our model, consumption of fossil fuel in current periods increases due to improvements in renewable technology that will make consumption of fossil fuel in the future less advantageous.

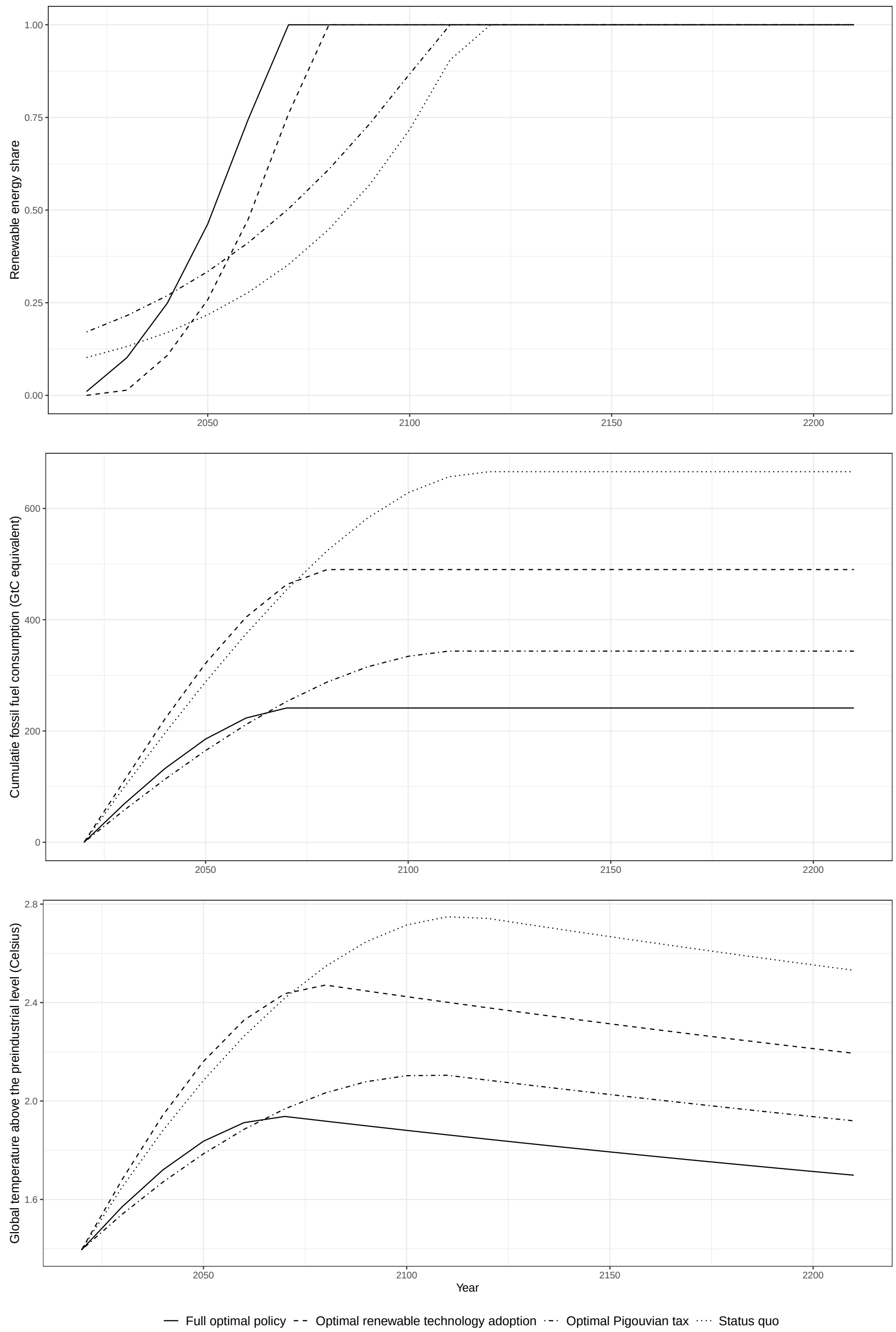

Figure 1: Benchmark Calibration: Energy and Temperature

flat.[26]

The third panel shows the path for global temperatures under our four policy scenarios. In order to map carbon concentrations into global temperatures, T, we use the following expression (see GHKT, 2014):

$$T(S_t) = 3\ln\left(\frac{S_t}{\overline{S}}\right) / \ln(2),$$

where $\overline{S}$ is the pre-industrial level of atmospheric carbon concentration. Consistent with the fossil fuel use in the top panel, the global temperature increases under both the business-as-usual and the optimal technology adoption scenarios, reaching short of 2.8 degrees Celsius above the pre-industrial level. The temperature under the technology policy alone (in the absence of a Pigouvian tax) later falls slightly faster than in the benchmark case. Under the optimal Pigouvian tax and in the fully optimal case, global temperatures peak at around 2.2 and 2.0 degrees Celsius above the pre-industrial level, respectively, and then decline over time.

GHKT (2014) find a peak temperature of almost 10 degrees Celsius above the pre-industrial level without policy intervention. In the context of the model, no policy intervention means that the price of fossil fuel is equal to the marginal cost of fossil fuel, which makes fossil fuel consumption quite attractive. The high temperature increase is explained by the fact that GHKT (2014) assume a larger total endowment of fossil fuel, zero carbon taxes, and no endogenous growth in productivity of renewable energy.[27] Instead, they assume that the extraction efficiency and the efficiency of green technologies grow both at the same rate (2 percent per year). The equal growth in productivity does not allow a productivity gain of renewables over fossil fuel over time, and contributes to the large increase in the temperature in their analysis. Additionally, GHKT (2014) find that the optimal tax would limit warming to about 3 degrees Celsius. This temperature rise is similar to the one obtained in the status quo of our calibrated model. This is due to the fact that in the status quo the renewable sector exhibits faster technological progress than the fossil fuel sector, and the carbon tax is already at almost at 2/3s of the damages resulting from carbon emissions. In the status quo case the renewable efficiency grows sufficiently fast so that the economy is fully renewable by 2120. In contrast, as can be seen in Figure 7 of GHKT (2014), they find that the economy continues to use fossil fuel even under the optimal case way beyond 2120.

The three panels in Figure 2 describe the over-time contribution of certain key variables to related growth rates under our four policy scenarios. The top panel shows the period-by-period difference in related damages caused by GHG emissions. Of note, the implementation of the technology policy in the absence

[26]GHKT (2014) assume an unbounded stock of coal reserves. This assumption simplifies their computation of the equilibrium. However, the resulting path for coal consumption under laissez-faire becomes implausible. In their own words: "under laissez-faire, coal use increases quickly, leading to a scarcity rent." Their model implies a coal consumption curve having a steep slope 200 years from now, and a value of approximately 180 GtC per year.

[27]They assume finite oil reserves but infinite coal reserves.

of a carbon tax has a greater negative effect on growth compared to the status quo. After the full transition takes place, the contribution to growth is positive (if small) in all four cases, due to the gradual decline in the stock of emissions. The second panel plots the contribution of the energy sector to economic growth. The status quo scenario results in a sizable negative contribution to growth. This is due partly to damages and partly to scarcity and the resulting increase in the shadow price of fossil fuel. As the resource constraint on fossil fuel is far from binding under the fully optimal policy scenario, equation (68) in the Appendix implies that the net contribution of energy to growth is positive and increasing during the energy transition.

Next, we turn our attention to welfare comparisons across these scenarios. Following Lucas (1987), we report the consumption-equivalent percentage welfare gain from these policies over the business-as-usual benchmark. Moving from the status quo to optimal technology adoption alone (in the absence of a Pigouvian tax) would imply a 0.251 percent consumption-equivalent gain, while the optimal Pigouvian tax alone would result in a gain of 1.023 percent, confirming the relative importance of the carbon tax. Comparing the status quo to the scenario where both policies are implemented results in a consumption-equivalent welfare gain of 1.431 percent.[28] This is one of the main findings of our quantitative analysis. The difference between the welfare gain from applying either policy in isolation versus implementing both amounts to about a 0.157 percent increase in consumption, suggesting a sizeable complementary between the two policies. Thus, our model points to sizable welfare gains only when the policies are adopted in tandem.

To further highlight the role of the capital replacement costs in these findings, we re-calibrate the model's status quo policy parameter under the same parametrization for preferences, climate factors, and technology the same as before, but with capital replacement costs "shut down"; i.e., setting $\Psi = 1$. In the absence of such costs, we set the growth rate in the renewable technology equal to its long-run value, as implied by the balanced growth path.[29] We then target $f_0 = 100\ GtC$, which implies $s_0 - s_{-1} = 2.1$ percent. The resulting Pigouvian tax rate is close to the one under technology adoption: $\tau^f = 0.62 \cdot \tau^*$. The implied dynamics for the share of renewables, fossil fuel consumption, and global temperatures, as well as the corresponding effects on growth, are reported in Figures 3 and 4. By comparing the cases with and without technology adoption costs, we notice a number of important differences. Optimal penetration by renewables starts lower in the case with replacement costs, but it soon overtakes, and the transition to the fully renewable state occurs earlier in this case. Fossil fuel consumption and global temperatures exhibit corresponding differences.

[28] Compared with other macroeconomic costs, like the costs of business-cycles or the costs of inflation, these are relatively large costs. Needless to say, these costs could be even higher given the uncertainties associated with the parametrization of economic damages from climate change and the climate sensitivity.

[29] Note that if, unlike in our benchmark case, the cost of adopting new renewable technologies is not proportional to output, then the optimal technological progress in the renewable sector would be similar to the case with constant long-run growth, as studied here.

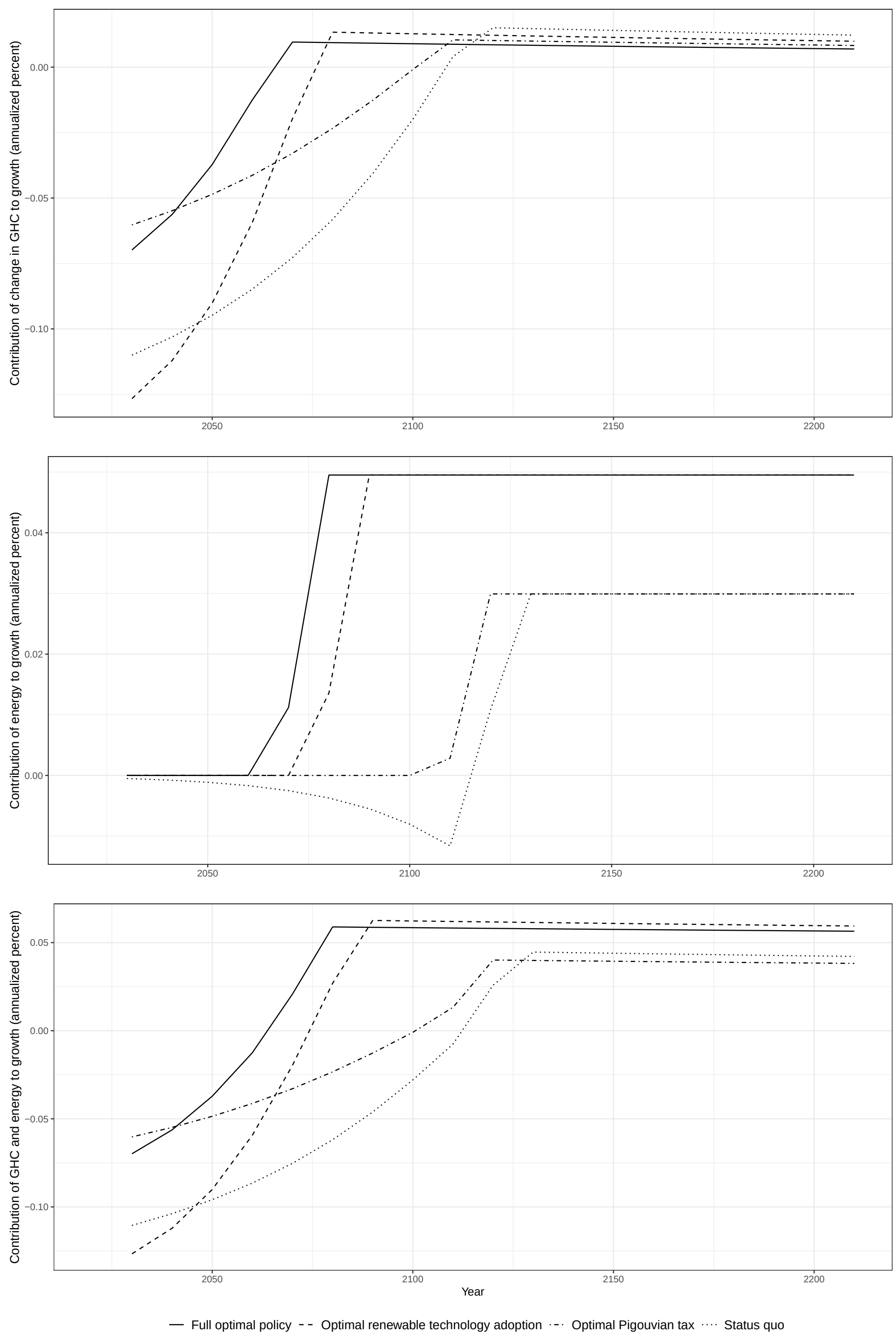

Figure 2: Benchmark Calibration: Growth Contributions

In the case without replacement costs, the consumption-equivalent welfare gain from setting the Pigouvian tax to its fully optimal level corresponds to a 1.05 percent increase in consumption. We conclude that introducing renewable technology adoption costs plays a non-negligible quantitative role in the optimal energy transition, as the difference between the welfare gain from applying the fully optimal policy with versus without these costs amounts to an approximate 0.381% increase in consumption.

The comparison between the benchmark case and the case without replacement costs also illustrates the complementarity between the two policies in our benchmark case. Because in the benchmark case the cost of renewable technology improvements is proportional to the capital deployed in the renewable sector, it is more effective in improving renewable technology in the case where a smaller share of energy is produced by the renewable sector. Thus, under the optimal renewable technology adoption, it is optimal to have a lower share of renewable early on and to catch up later. If instead the cost of improving renewable technology was independent of the capital deployed in this sector, the technological improvement rate in the renewable sector would be constant and independent of the share of renewables in energy production. Therefore, the case where adoption costs are not commensurate with the amount of capital used in the renewable sector is similar to the case without replacement costs that we described above. In such a case, adopting the optimal Pigouvian tax would not affect the dynamics of improvement in the renewable technology. Thus, there would be no complementarity between the optimal renewable technology adoption policy and the optimal Pigouvian tax. In contrast, in our benchmark case where the cost of improvements in renewable technology is proportional to the capital used in the sector, the optimal Pigouvian tax reduces the initial negative effect from the optimal renewable adoption policy. In summary, the dynamics of the improvements in renewable energy resulting from capital replacement costs contribute towards a complementarity between the optimal technology adoption policy and the optimal Pigouvian tax.

6.1 Productivity Growth in the Fossil Fuel Sector

As a robustness check, here we explore how our findings are affected if we consider productivity growth in the fossil fuel sector. We calibrate this to match the actual growth rate between 1950 and 2000. We impose two additional restrictions. First, we assume a constant share of energy in the global economy during these five decades. Second, we assume that renewable energy was a small share of total energy, and that almost the entire increase in energy production during that period was due to the rise in the use of fossil fuel. Between 1950 and 2000, the annualized growth rate of world GDP in constant prices was 1.68 percent. In the same period, the annualized growth rate of CO_2 emissions was 1.27 percent.[30] Based on these observations, and assuming a constant

[30] See https://www.statista.com/statistics/264699/worldwide-co2-emissions/ and https://ourworldindata.org/grapher/world-gdp-over-the-last-two-millennia.

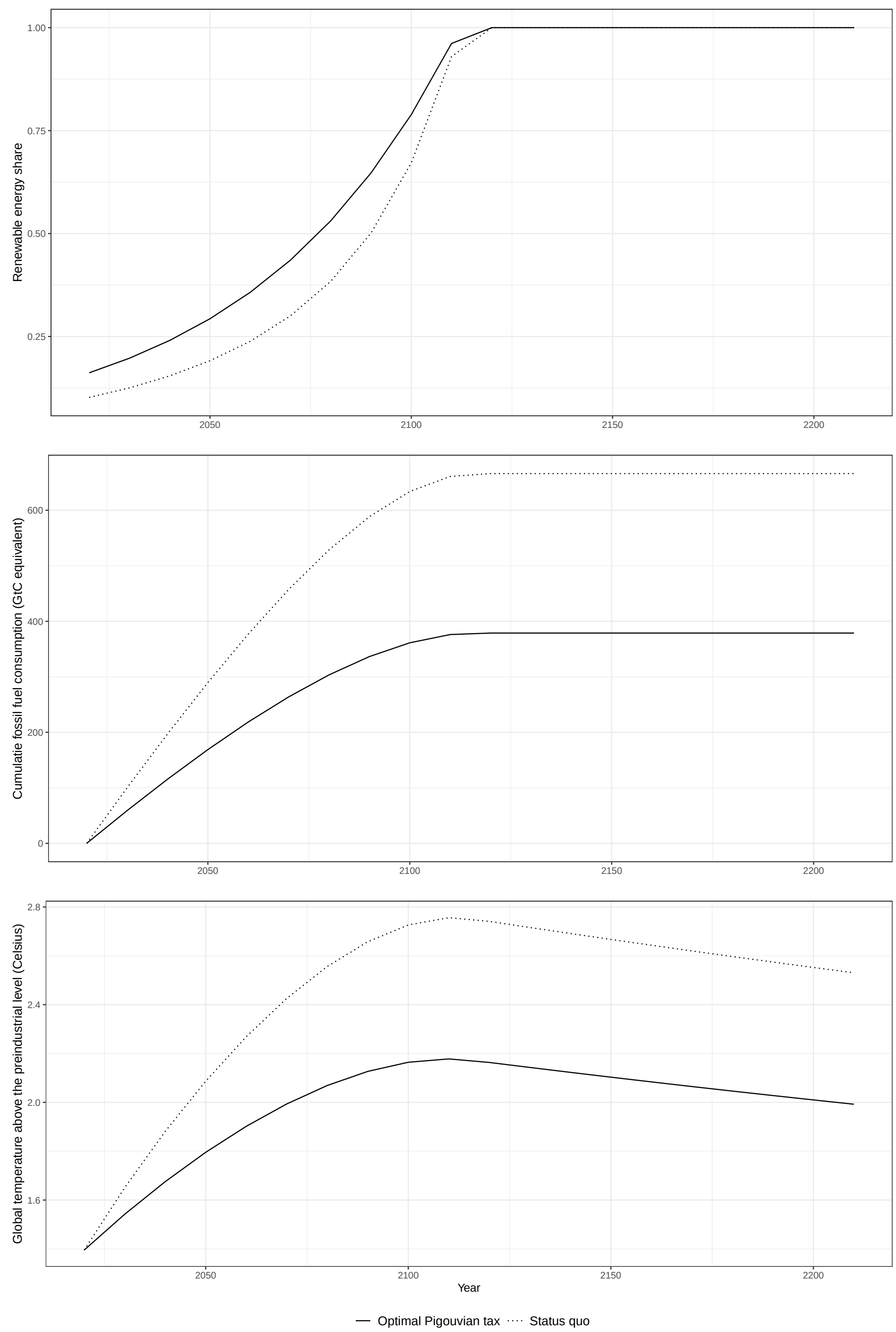

Figure 3: No Scrapping: Energy and Temperature

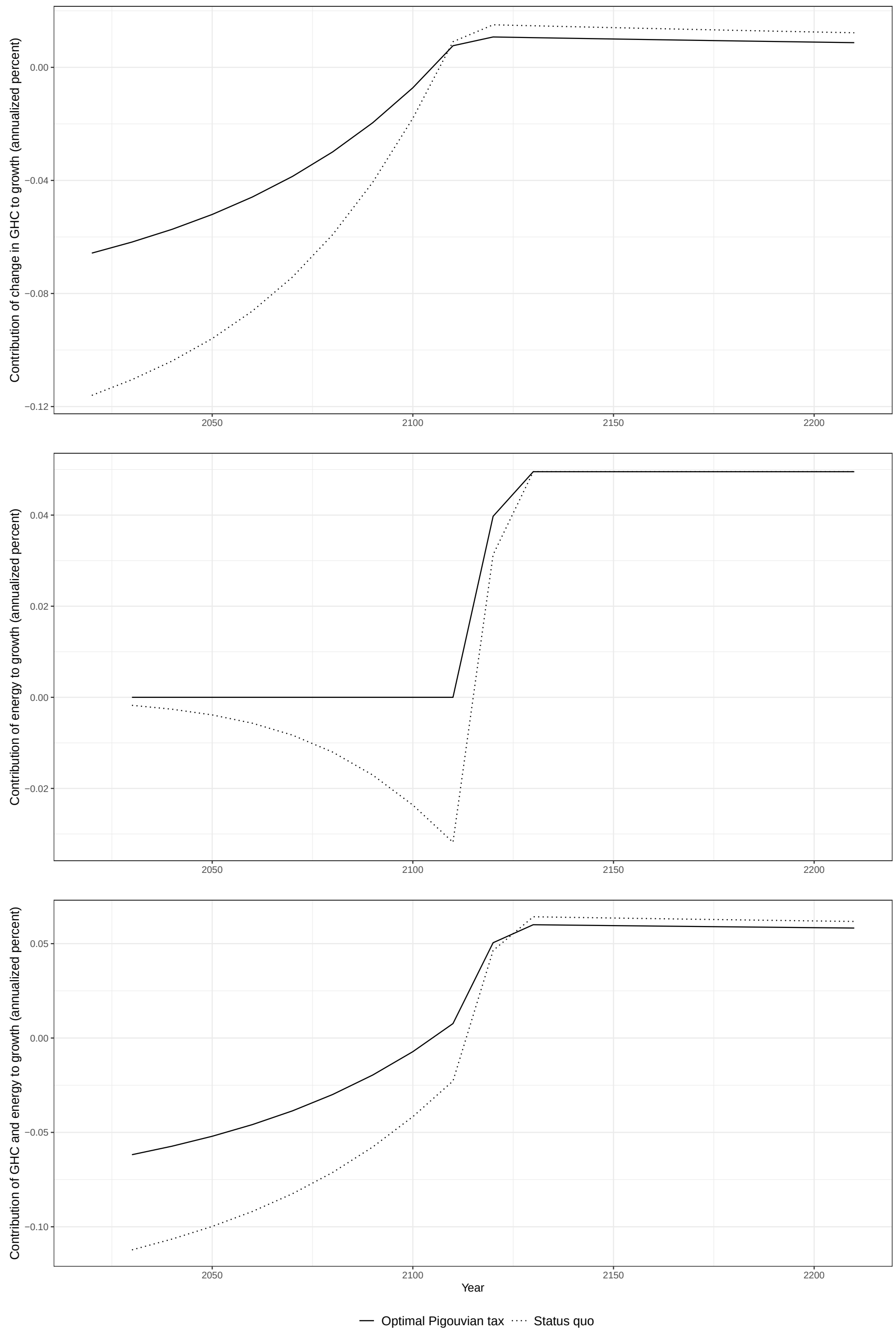

Figure 4: No Scrapping: Growth Contribution

carbon intensity, the productivity of the fossil fuel sector has grown at the annualized rate of 0.40 percent during that period.

Incorporating productivity growth in fossil fuel changes the calibrated values of both the spillover parameter, ξ, and the Pigouvian tax rate, τ^f, relative to the benchmark calibration. The biggest difference is in the new calibrated value of ξ, which is closer to the optimal level at $\xi = 0.69$ compared to $\xi = 0.54$ in the benchmark calibration. This is because the renewable sector must now exhibit higher productivity growth relative to the benchmark in order to match the same level of increase in the share of renewables. In contrast, the calibrated value for the Pigouvian tax rate remains at $\tau^f = 0.63 \cdot \tau^*$, almost the same as its benchmark calibrated level.

Figure 5 shows the paths for the share of renewable energy (top), accumulated fossil fuel consumption (middle), and global temperatures (bottom), under our four scenarios. Like in Figure 1, these scenarios correspond to the status quo (dotted line), optimal renewable technology adoption (dashed line), optimal Pigouvian carbon taxation (dot-dashed line), and combined optimal policies (solid line).

A comparison between Figures 1 and 5 reveals that the economy reaches the full renewable stage earlier when we incorporate productivity growth in the fossil fuel sector to the model. This seemingly counter-intuitive result is due to the difference in the calibrated value of ξ, as the spillover from renewable technology adoption is higher when we incorporate productivity growth in fossil fuel. As a result, the productivity of the renewable sector grows faster and surpasses earlier the level at which the fossil fuel sector is no longer competitive. In addition, as the middle panel of Figure 5 demonstrates, fossil fuel is not exhausted under the status quo path. Consequently, the maximum global temperature reaches 2.6 degrees Celsius above the pre-industrial level under the status quo (Figure 5 bottom panel), below the 2.8 degrees Celsius achieved under the status quo path of the benchmark calibration (Figure 1).

The qualitative effects of switching from the status quo to the paths associated with optimal renewable technology adoption and optimal Pigouvian taxation, respectively, are similar to those in our benchmark calibration. As seen in the top panel of Figure 5, the optimal renewable technology adoption path results in an immediate lower renewable share. However, faster growth in the renewable sector results in an earlier transition to a fully renewable economy. In contrast, switching from the status quo to the optimal Pigouvian taxation path would result in an immediate rise in the renewable share, but the path for renewables in this case is more or less parallel to the status quo. Finally, the combined optimal policy leaves the current level of renewable energy share more or less unchanged, but the resulting path in this case would be parallel to the optional adoption path, thus reaching the fully renewable economy faster than in the other cases. The paths for fossil fuel consumption and for global temperatures (middle and bottom panels, respectively, of Figure 5) remain similar to those in the benchmark calibration (middle and bottom panels, respectively, of Figure 1).

Moving from the status quo to optimal technology adoption alone (but no

Pigouvian tax) would imply a 0.74 percent consumption-equivalent gain, while the optimal Pigouvian tax alone would result in a gain of 1.09 percent, confirming the relative importance of the carbon tax. Comparing the status quo to the situation where both policies are applied results in a consumption-equivalent welfare gain of 1.53 percent. These findings are broadly consistent with the welfare gains in our benchmark case.

7 Conclusion

We incorporated costs associated with adopting new capital in the renewable energy sector in an IAM framework based on Golosov et al (2014). In a capital-intensive industry like the energy sector, when technological progress is embedded in the capital stock, such costs can have sizable quantitative implications for the optimal share of renewable energy, especially in the short run. We investigated their quantitative implications for the optimal energy transition. Of course, the details of modeling these costs become less relevant on a balanced growth path, when renewables become the dominant energy source. We found that in the case where Pigouvian carbon taxes are infeasible, tax/subsidy policies that subsidize renewables (in our case by internalizing spillover effects) might not be a suitable substitute. Hassler et al. (2020) found that, when it comes to accomplishing climate-related goals, making renewable energy cheaper is not necessarily an effective substitute for making fossil fuel more expensive. Our model reaches a similar conclusion. This suggests that these two policies are better thought of as complements. The tax/subsidy scheme forces firms to incur higher technology adoption costs, for example, by scrapping a larger portion of their capital than under the status quo path. Particularly in early years, when the policy has not yet sufficiently altered the productivity of the renewable sector relative to the status quo, this discourages heavy investment in renewable energy capital. As time passes, and the policy results in a significantly more productive renewable sector, it becomes more efficient to invest more heavily in renewables, resulting in their share more quickly surpassing that of fossil fuel. We found this conclusion to hold even when we incorporate technological progress in fossil fuel into the model.

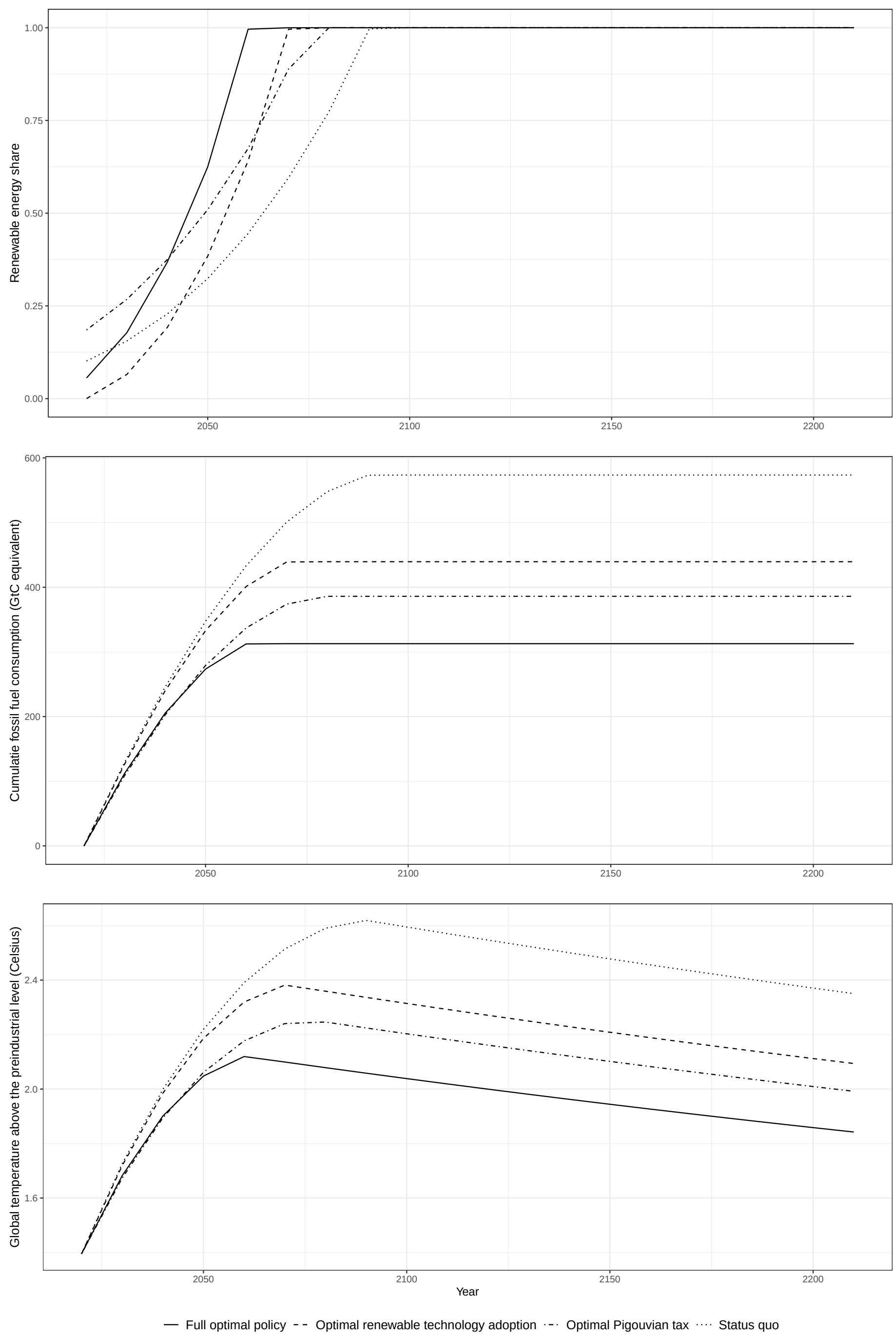

Figure 5: Long-run Economy with Productivity Growth in Fossil Fuel

8 Appendix

8.1 Optimization by Households and Firms

The representative household owns the firms as well as the capital and fossil fuel stocks. It rents capital to firms and sells fossil fuel to the non-renewable sector. The representative household's problem is given by

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{s.t.}$$

$$\sum_{t=0}^{\infty} p_t \left[c_t + k_{t+1} - (1-\delta)k_t \right] \leq$$

$$\sum_{t=0}^{\infty} p_t \left[r_t k_t + w_t l_t + p_t^f f_t + \int_0^1 \pi_{j,t}^r dj + z_t \right],$$

$$\varpi_{t+1} \leq \varpi_t - f_t, \tag{25}$$

where p_t is the Arrow-Debreu price of the period t final good, δ is the depreciation rate of capital, r_t is the rental price of capital, w_t is the wage rate, p_t^f is the price of fossil fuel, $\int_0^1 \pi_{j,t}^r dj$ stands for the profits of renewable firms, and z_t are lump-sum transfers from the government. The government collects taxes from the fossil fuel energy sector and rebates them lump-sum to households, balancing its budget in every period.

The FOCs, which are also sufficient for a maximum, imply

$$1 - \delta + r_{t+1} = \frac{\beta u'(c_{t+1})}{u'(c_t)} \tag{26}$$

and

$$\frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{p_{t+1}}{p_t}. \tag{27}$$

Equation (26) says that the rental price of capital plus the non-depreciated part of capital must equal the marginal rate of substitution between consumption in two consecutive periods. Equation (27) says that the marginal rate of substitution between consumption in period t and consumption in period $t+1$ must equal the relative price of the respective consumption goods.

The final-good-producing firms rent capital, hire labor, and buy energy in competitive markets at prices w_t, r_t, and p_t^e, respectively. The representative firm in the final good sector solves

$$\max \left[A_t \cdot (k_t^c)^{\theta_k} (l_t)^{\theta_l} (e_t)^{1-\theta} - r_t k_t^c - w_t l_t - p_t^e e_t \right].$$

The FOCs imply that the marginal input productivities equal their respective prices:

$$\theta_k A_t (k_t^c)^{\theta_k - 1} (l_t)^{\theta_l} (e_t)^{1-\theta} = r_t, \tag{28}$$

$$\theta_l A_t \left(k_t^c\right)^{\theta_k} \left(l_t\right)^{\theta_l - 1} \left(e_t\right)^{1-\theta} = w_t, \tag{29}$$

and

$$\left(1-\theta\right) A_t \frac{\left(k_t^c\right)^{\theta_k} \left(l_t\right)^{\theta_l}}{e_t^{\theta}} = p_t^e. \tag{30}$$

Firms in the fossil fuel sector rent capital and buy fossil fuel. Additionally, they pay a per unit tax on the GHG emission from fossil fuel use, τ_t. The representative firm in this sector solves

$$\max \left[p_t^e A_f \left(f_t\right)^{1-\alpha_f} \left(k_t^f\right)^{\alpha_f} - r_t k_t^f - \left(p_t^f + \tau_t\right) f_t\right].$$

The FOCs imply that the value of the marginal input productivities equal their respective prices:

$$p_t^e \alpha_f A_f \left(\frac{f_t}{k_t^f}\right)^{1-\alpha_f} = r_t \tag{31}$$

and

$$p_t^e \left(1-\alpha_f\right) A_f \left(\frac{k_t^f}{f_t}\right)^{\alpha_f} = \left(p_t^f + \tau_t\right). \tag{32}$$

The production function for the renewable energy firms is given by (12). It depends on the firm's productivity, the firm's technology adoption rate, and the capital used. The firms in this sector rent capital and receive a subsidy, $\Phi\left(i_{j,t}\right)$, which is a function of the firm's technology adoption rate, $i_{j,t}$. We allow $\Phi\left(i_{j,t}\right)$ to be negative and assume it is differentiable. In each period t, the renewable firm j maximizes future discounted profits subject to (13):

$$\begin{aligned} \max \sum_{\tau=0}^{\infty} \beta^{t+\tau} u'(c_{t+\tau}) & \left[p_{t+\tau}^e \Psi(i_{j,t+\tau}) \left(\mathcal{E}_{j,t+\tau}\right)^{1-\alpha_r} \left(k_{j,t+\tau}^r\right)^{\alpha_r} - r_{t+\tau} k_{j,t+\tau}^r + \Phi\left(i_{j,t+\tau}\right)\right] \\ \text{s.t. } \ln \mathcal{E}_{t+1}^j & \leq \ln \mathcal{E}_t^j + \xi i_{j,t} + (1-\xi) \left(\int_0^1 i_{j,t} k_{j,t}^r dj \Big/ \int_0^1 k_{j,t}^r dj\right) \\ & i_{j,t} \geq 0, \text{ and } \mathcal{E}_0 \text{ given.} \end{aligned} \tag{33}$$

Let $\lambda_{\mathcal{E},t}^j$ be the Lagrangian multiplier associated with equation (13). The FOCs of this problem are

$$p_t^e \alpha_r \Psi(i_{j,t}) \left(\frac{\mathcal{E}_{j,t}}{k_{j,t}^r}\right)^{1-\alpha_r} = r_t, \tag{34}$$

$$-\beta^t u'(c_t) \left[p_t^e \Psi'(i_{j,t}) \left(\mathcal{E}_{j,t}\right)^{1-\alpha_r} \left(k_{j,t}^r\right)^{\alpha_r} + \Phi'\left(i_{j,t}\right)\right] = \xi \lambda_{\mathcal{E},t}^j, \tag{35}$$

and

$$\lambda_{\mathcal{E},t+1}^j + \beta^{t+1} u'(c_{t+1}) p_{t+1}^e (1-\alpha_r) e_{j,t+1}^r = \lambda_{\mathcal{E},t}^j. \tag{36}$$

Equation (34) says that the value of the marginal productivity of capital should be equal to its rental price. Equation (35) says that the cost of increasing the adoption rate, which is the loss in production plus the marginal subsidy, should equal the benefit from increasing the adoption rate, which comes from the value of having a higher level of productivity next period. Equation (36) says that the value this period from relaxing constraint (13) should be equal to the value from relaxing that constraint next period plus the benefit of higher productivity next period.

8.2 Proof of Proposition 2

Proposition 2 in the text states:

Proposition 2: *In a competitive equilibrium with $\Phi\left(i_{j,t}\right)=0$, $i_{j,t}$ is lower than optimal when $\xi<1$.*

Proof. From Proposition 1, the social planner chooses $i_{j,t}=i_t$ and $\frac{k^r_{j,t}}{\mathcal{E}_{j,t}}=\frac{k^r_t}{\mathcal{E}_t}$. This, together with the FOC (20), implies that

$$-\Psi'(i_t)\mathcal{E}_{j,t}\left(\frac{k^r_t}{\mathcal{E}_t}\right)^{\alpha_r}\mu^j_{r,t}=\xi\mu^j_{\mathcal{E},t}+(1-\xi)\frac{k^r_{j,t}}{\overline{k^r_t}}\int_0^1\mu^j_{\mathcal{E},t}dj. \tag{37}$$

The FOCs of the social planer's problem also give

$$\beta^t u'(c_t)\left(1-\theta\right)A_t\frac{\left(k^c_t\right)^{\theta_k}\left(L_t\right)^{\theta_L}}{\left(e_t\right)^{\theta}}=\mu_{E,t}=\mu^j_{r,t}. \tag{38}$$

Equation (37) together with (38) and (30), implies

$$-\beta^t u'(c_t)p^e_t\Psi'(i_t)\mathcal{E}_{j,t}\left(\frac{k^r_t}{\mathcal{E}_t}\right)^{\alpha_r}=\xi\mu^j_{\mathcal{E},t}+(1-\xi)\frac{k^r_{j,t}}{\overline{k^r_t}}\int_0^1\mu^j_{\mathcal{E},t}dj. \tag{39}$$

The FOC with respect to $\mathcal{E}_{j,t+1}$ is

$$\mu^j_{\mathcal{E},t+1}\frac{1}{\mathcal{E}^j_{t+1}}+\mu^j_{r,t+1}(1-\alpha_r)\Psi(i_{j,t+1})\left(\frac{k^r_{j,t+1}}{\mathcal{E}_{j,t+1}}\right)^{\alpha_r}=\mu^j_{\mathcal{E},t}\frac{1}{\mathcal{E}^j_{t+1}}, \tag{40}$$

which can be rewritten using condition (12) as

$$\mu^j_{\mathcal{E},t+1}+\beta^{t+1}u'(c_{t+1})p^e_{t+1}(1-\alpha_r)e^r_{j,t+1}=\mu^j_{\mathcal{E},t}.$$

Solving for $\mu^j_{\mathcal{E},t}$, we obtain

$$\mu^j_{\mathcal{E},t}=\sum_{\tau=1}^{\infty}\beta^{t+\tau}u'(c_{t+\tau})p^e_{t+\tau}(1-\alpha_r)e^r_{j,t+\tau},\ \text{if}\ \lim_{\tau\to\infty}\mu^j_{\mathcal{E},\tau}=0. \tag{41}$$

Replacing (41) in (39), we obtain

$$-\Psi'(i_t)\mathcal{E}_{j,t}\left(\frac{k_t^r}{\mathcal{E}_t}\right)^{\alpha_r} = \xi\sum_{\tau=1}^{\infty}\beta^{\tau}\frac{u'(c_{t+\tau})p_{t+\tau}^e}{u'(c_t)p_t^e}(1-\alpha_r)e_{j,t+\tau}^r + (1-\xi)\sum_{\tau=1}^{\infty}\beta^{\tau}\frac{u'(c_{t+\tau})p_{t+\tau}^e}{u'(c_t)p_t^e}(1-\alpha_r)\frac{k_{j,t+\tau}^r}{\overline{k_{t+\tau}^r}}\int_0^1 e_{j,t+\tau}^r dj. \quad (42)$$

Solving equation (36) for $\lambda_{\mathcal{E},t}^j$, we obtain

$$\lambda_{\mathcal{E},t}^j = \sum_{\tau=1}^{\infty}\beta^{t+\tau}u'(c_{t+\tau})p_{t+\tau}^e(1-\alpha_r)e_{j,t+\tau}^r, \text{ if } \lim_{\tau\to\infty}\lambda_{\mathcal{E},\tau}^j = 0. \quad (43)$$

Finally, replacing (43) in equation (35) (with $\Phi(i_{j,t}) = 0$) gives

$$-\Psi'(i_{j,t})(\mathcal{E}_{j,t})\left(\frac{k_{j,t}^r}{\mathcal{E}_{j,t}}\right)^{\alpha_r} = \xi\sum_{\tau=1}^{\infty}\beta^{\tau}\frac{u'(c_{t+\tau})p_{t+\tau}^e}{u'(c_t)p_t^e}(1-\alpha_r)e_{j,t+\tau}^r. \quad (44)$$

It is straightforward to verify that the right-hand side of equation (42) is larger than the right-hand side of equation (44). Since $-\Psi'(i_{j,t})$ is increasing in $i_{j,t}$, everything else being equal, the value of $i_{j,t}$ that satisfies (44) in the competitive equilibrium equation is lower than the i_t that satisfies (42) in the social planner's FOC. ■

8.3 Proof of Proposition 3

Proposition 3 in the text states:

Proposition 3: (1) *The optimal allocation can be supported by a combination of a revenue-neutral policy,* $\Phi(i_{j,t}) = -(1-\xi)p_t^e\Psi'(i_t^*)\left(\frac{e_{j,t}^{*r}}{\Psi(i_{j,t}^*)}\right)(i_{j,t}-i_t^*)$, imposed on renewable firms, *together with a Pigouvian tax on fossil fuel use,*
$\tau_t^f = \sum_{j=0}^{\infty}\beta^j\frac{u'(c_{t+j}^*)}{u'(c_t^*)}\pi_{t+j}y_{t+j}^*(1-d_j)$,
where $\{c_t^*, y_t^*, i_t^*\}_{t=0}^{\infty}$ *is the solution to the planner's problem, and* $1-d_j = \varphi_L + (1-\varphi_L)\varphi_0(1-\varphi)^j$. *(2) If* $u(c) = \log(c)$, $\alpha_r = \alpha_f = \alpha$, $\pi_t = \pi$, all t, *and* $\delta = 1$, $\tau_t^f = y_t\pi\left[\frac{\varphi_L}{1-\beta} + \frac{(1-\varphi_L)\varphi_0}{1-(1-\varphi)\beta}\right]$ *grows at the growth rate of the economy.*

Proof. When $\Phi(i_{j,t}) = -(1-\xi)p_t^e\Psi'(i_t^*)\left(\frac{e_{j,t}^{*r}}{\Psi(i_{j,t}^*)}\right)(i_{j,t}-i_t^*)$, the firm j's FOC (35) is

$$-\beta^t u'(c_t)\left[p_t^e\Psi'(i_t^*)\left(\frac{e_{j,t}^{*r}}{\Psi(i_{j,t}^*)}\right)\right] = \lambda_{\mathcal{E},t}^j. \quad (45)$$

Solving (36) for $\lambda_{\mathcal{E},t}^j$, we obtain

$$\lambda_{\mathcal{E},t}^j = \sum_{\tau=1}^{\infty}\beta^{t+\tau}u'(c_{t+\tau})p_{t+\tau}^e(1-\alpha_r)e_{j,t+\tau}^r, \text{ if } \lim_{\tau\to\infty}\lambda_{\mathcal{E},\tau}^j = 0. \quad (46)$$

Combining this equation with (45) gives

$$-\Psi'(i_t^*)\left(\mathcal{E}_{j,t}\right)\left(\frac{k_t^r}{\mathcal{E}_t}\right)^{\alpha_r} = \sum_{\tau=1}^{\infty}\beta^\tau \frac{u'(c_{t+\tau})p_{t+\tau}^e}{u'(c_t)p_t^e}(1-\alpha_r)e_{j,t+\tau}^r. \quad (47)$$

The social planner's problem gives rise to a similar condition, (42), which we repeat here:

$$\begin{aligned}-\Psi'(i_t)\mathcal{E}_{j,t}\left(\frac{k_t^r}{\mathcal{E}_t}\right)^{\alpha_r} &= \xi\sum_{\tau=1}^{\infty}\beta^\tau \frac{u'(c_{t+\tau})p_{t+\tau}^e}{u'(c_t)p_t^e}(1-\alpha_r)e_{j,t+\tau}^r + (1-\xi) \\ &\quad \sum_{\tau=1}^{\infty}\beta^\tau \frac{u'(c_{t+\tau})p_{t+\tau}^e}{u'(c_t)p_t^e}(1-\alpha_r)\frac{k_{j,t+\tau}^r}{\overline{k_{t+\tau}^r}}\int_0^1 e_{j,t+\tau}^r dj \quad (48)\end{aligned}$$

To show that these conditions are identical, thus implying the same i_t, it suffices to show that

$$\frac{k_{j,t}^r}{\overline{k_t^r}}\int_0^1 e_{j,t}^r dj = e_{j,t}^r. \quad (49)$$

This follows from

$$\begin{aligned}\frac{k_{j,t}^r}{\overline{k_t^r}}\int_0^1 e_{j,t}^r dj &= \frac{k_{j,t}^r}{\int_0^1 k_{j,t}^r dj}\int_0^1 \Psi(i_t)k_{j,t}^r\left(\frac{k_t^r}{\mathcal{E}_t}\right)^{\alpha_r-1} dj \\ &= \frac{k_{j,t}^r}{\int_0^1 k_{j,t}^r dj}\Psi(i_t)\left(\frac{k_t^r}{\mathcal{E}_t}\right)^{\alpha_r-1}\int_0^1 k_{j,t}^r dj \\ &= k_{j,t}^r\Psi(i_t)\left(\frac{k_t^r}{\mathcal{E}_t}\right)^{\alpha_r-1} = e_{j,t}^r. \quad (50)\end{aligned}$$

Next, suppose that sellers of fossil fuel face a linear tax rate,

$$\tau_t^f = \sum_{j=0}^{\infty}\beta^j\frac{u'(c_{t+j}^*)}{u'(c_t^*)}\pi_{t+j}y_{t+j}^*(1-d_j), \quad (51)$$

where $\{c_t^*, y_t^*\}_{t=0}^{\infty}$ solves the planner's problem, and $1-d_j = \varphi_L + (1-\varphi_L)\varphi_0(1-\varphi)^j$. Under this tax, the fossil fuel producers' optimal intertemporal substitution implies

$$u'(c_t)\cdot p_t^f = \beta u'(c_{t+1})\cdot p_{t+1}^f. \quad (52)$$

Using (32) for the price of fossil fuel, we obtain

$$u'(c_t)\left\{MPF_t - \tau_t^f\right\} = \beta u'(c_{t+1})\left\{MPF_{t+1} - \tau_{t+1}^f\right\},$$

and using (51) for the tax, we obtain

$$\begin{aligned}&u'(c_t)\left\{MPF_t - \pi_t y_t^*\left(\varphi_L + (1-\varphi_L)\varphi_0\right)\right\} \\ &+\sum_{j=1}^{\infty}\beta^j u'(c_{t+j}^*)\pi_{t+j}y_{t+j}^*((1-\varphi_L)\varphi_0(1-\varphi)^{j-1}\varphi) \\ = \ &\beta u'(c_{t+1})\left\{MPF_{t+1}\right\}, \quad (53)\end{aligned}$$

where MPF_t is the period t marginal productivity of fossil fuel in units of the final good. Clearly, the claim follows if $\frac{y^*_{t+j}}{c^*_{t+1}} = \chi$, a constant. First, observe that $\frac{c_t}{y_t} = \chi \Leftrightarrow \frac{k^g_{t+1}}{y_t} = \theta^k \beta$. This equation follows from the FOCs of the social planner, which include

$$\frac{y_t}{c_t} = \frac{y_{t+1}}{c_{t+1}} \frac{\theta^k \beta y_t}{k^g_{t+1}}. \tag{54}$$

It remains to be shown that

$$\frac{k^f_{t+1}}{y_t} + \frac{k^r_{t+1}}{y_t} = 1 - \chi - \theta^k \beta \equiv \varrho, \tag{55}$$

where $k^r_t = \int k^r_{t,m} dm$. The social planner problem's FOCs with respect to $k^r_{j,t}$ implies

$$\alpha_r \Psi(i_t) \left(\frac{\mathcal{E}_{j,t}}{k^r_{j,t}} \right)^{1-\alpha_r} (1-\theta) \frac{y_t}{e_t} = \alpha_r \left(\frac{e^r_{j,t}}{k^r_{j,t}} \right) (1-\theta) \frac{y_t}{e_t} = \theta^k \frac{y_t}{k^g_t} \tag{56}$$

$$\alpha_r (1-\theta) \frac{e^r_{j,t}}{e_t} = \frac{1}{\beta} \frac{k^r_{j,t}}{y_{t-1}}. \tag{57}$$

The FOC with respect to k^f_t implies

$$\alpha_f (1-\theta) \frac{e^f_t}{e_t} = \frac{1}{\beta} \frac{k^f_t}{y_{t-1}}. \tag{58}$$

It is sufficient to show that

$$\beta (1-\theta) \left(\frac{\alpha_r e^r_{t+1} + \alpha_f e^f_{t+1}}{e_{t+1}} \right) = \varrho, \tag{59}$$

which is true if $\alpha_r = \alpha_f = \alpha$. ■

8.4 The Optimal Transition

Here we characterize the equilibrium allocation across the transition, and we derive some key expressions that are used in our calibration. Let $V(k; A, L, \mathcal{E}, w; \Gamma^p, \Gamma^d)$ denote the value given k available units of capital and given that the aggregate productivity is A, the labor supply is l, the productivity in the renewable energy sector is $\mathcal{E}$, the stock of fossil fuel is w, and the stocks of permanent and depreciating emissions are Γ^p and Γ^d, respectively. We let g stand for the percentage productivity growth rate in the final good sector, while g^l is the population growth rate.

The optimal consumption and saving decision under log utility and full depreciation is given by $c = (1-\beta\Theta)y$ and $k' = \beta\Theta y$, where $\Theta = \theta_k + (1-\theta_k-\theta_\ell)\alpha$

is the marginal product of capital. The recursive formulation for $V(\cdot)$ is given by

$$
\begin{aligned}
V\left(k; A, L, \mathcal{E}, w; \Gamma^p, \Gamma^d\right) \quad = \quad & \max_{i,f} \{\ln((1-\beta\Theta)y) \\
& +\beta V(\beta\Theta y; gA, g^l l, e^i\mathcal{E}, w-f; \Gamma^{p\prime}, \Gamma^{d\prime})\} \\
& \text{where} \\
& y = e^{-\pi(\Gamma^{p\prime}+\Gamma^{d\prime}-\bar{\Gamma})} A L^{\theta_\ell} \left(f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}\right)^{(1-\alpha)(1-\theta_k-\theta_\ell)} k^{\Theta} \\
& \Gamma^{p\prime} = \Gamma^p + \varphi_L f \\
& \Gamma^{d\prime} = (1-\varphi)\Gamma^d + (1-\varphi_L)\varphi_0 f. \qquad (60)
\end{aligned}
$$

Utilizing the envelope theorem, we have $V_k = \Theta\frac{1}{k} + \beta\Theta\frac{k'}{k}V'_{k'}$, which implies

$$kV_k = \Theta + \beta\Theta k' V_{k'}. \qquad (61)$$

We guess that kV_k is a constant and we verify that

$$V_k = \frac{\Theta}{1-\beta\Theta}\frac{1}{k}. \qquad (62)$$

Using the same method, we have that $V_A = \frac{1}{A} + \beta\left\{\frac{k'}{A}V'_{k'} + gV'_{A'}\right\}$, which, in turn, implies

$$AV_A = 1 + \beta\left\{\frac{\Theta}{1-\beta\Theta} + (gA)V'_{A'}\right\}. \qquad (63)$$

Next, we guess that AV_A is a constant. As $A' = gA$, this equation allows us to verify that

$$V_A = \frac{1}{(1-\beta)(1-\beta\Theta)}\frac{1}{A}. \qquad (64)$$

Similarly, we obtain

$$
\begin{aligned}
V_L &= \frac{\theta_l}{(1-\beta)(1-\beta\Theta)}\frac{1}{L}, && (65) \\
V_{\Gamma^p} &= \frac{1}{(1-\beta)(1-\beta\Theta)}(-\pi), && (66) \\
V_{\Gamma^d} &= \frac{1-\varphi}{(1-\beta(1-\varphi))(1-\beta\Theta)}(-\pi). && (67)
\end{aligned}
$$

The last expression reflects the depreciation rate of the temporary part of the emissions stock. Finally, the marginal value of stock of fossil fuel is given by

$$V_w = \beta \cdot V'_{w-f}.$$

The optimal choice of f on the equilibrium path implies

$$\frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{f+\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}(1+\beta\cdot k'V'_{k'}) \le V_w + \tau\left\{\begin{array}{l} \pi\cdot(\varphi_L + (1-\varphi_L)\varphi_0)(1+\beta\cdot k'V'_{k'}) \\ -\beta\left[\varphi_L V'_{\Gamma^{p\prime}} + (1-\varphi_L)\varphi_0 V'_{\Gamma^{d\prime}}\right] \end{array}\right\},$$

with equality when $f > 0$. The left-hand side of the above inequality gives the marginal benefit from consumption and from future capital accumulation, respectively. The first term of the right-hand side, $V_w = \beta V'_{w-f}$, is the price of fossil fuel. The second term is the tax on consumption of fossil fuel. Note that $\tau \in [0,1]$, so this tax could take any value from zero to the total value of the present and future damages resulting from GHG emissions. We will find it convenient to rewrite the above inequality as follows:

$$f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E} \geq \frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\tau\pi\left\{\frac{\varphi_L}{1-\beta} + \frac{(1-\varphi_L)\varphi_0}{1-\beta(1-\varphi)}\right\} + (1-\beta\Theta)V_w}, \tag{68}$$

with equality for $f > 0$. For $f, f' > 0$, using $V_w = \beta V'_{w'}$ and (68), we obtain

$$\left(f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}\right)^{-1} = \beta\left(f' + \Psi(i')^{\frac{1}{1-\alpha}}\mathcal{E}'\right)^{-1} + (1-\beta)\frac{\tau\pi\left\{\frac{\varphi_L}{1-\beta} + \frac{(1-\varphi_L)\varphi_0}{1-\beta(1-\varphi)}\right\}}{(1-\alpha)(1-\theta_k-\theta_\ell)}. \tag{69}$$

We use equation (69) to find the equilibrium path of fossil fuel consumption by solving this path backward. To do so, we also need to determine the equilibrium path of the renewable energy productivity.

The optimal choice for i, when the representative agent takes into account only ξ fraction of the benefit of higher i on future renewable productivity, implies

$$0 = (1-\alpha)(1-\theta_k-\theta_\ell)\frac{\frac{1}{1-\alpha}\Psi'(i)\Psi(i)^{\frac{1}{1-\alpha}-1}\mathcal{E}}{f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}\left\{1 + \beta\frac{\Theta}{1-\beta\Theta}\right\} + \beta\cdot\xi\underbrace{e^i\mathcal{E}}_{\mathcal{E}'}V'_{\mathcal{E}'} \tag{70}$$

or

$$\frac{-\Psi'(i)}{\Psi(i)}\frac{(1-\theta_k-\theta_\ell)}{1-\beta\Theta}\frac{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}{f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}} = \beta\cdot\xi\mathcal{E}'V'_{\mathcal{E}'}. \tag{71}$$

Utilizing the envelope theorem, we have

$$\mathcal{E}V_{\mathcal{E}} = \frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{1-\beta\Theta}\frac{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}{f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}} + \beta\mathcal{E}'V'_{\mathcal{E}'}. \tag{72}$$

Combining the above equation with (71) , we obtain

$$\frac{-\Psi'(i)}{\Psi(i)} = \beta\cdot\frac{\frac{\Psi(i')^{\frac{1}{1-\alpha}}\mathcal{E}'}{f' + \Psi(i')^{\frac{1}{1-\alpha}}\mathcal{E}'}}{\frac{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}{f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}}\left(\xi(1-\alpha) + \frac{-\Psi'(i')}{\Psi(i')}\right). \tag{73}$$

Equation (73) shows how the evolution of the elasticity of the technology adoption cost with respect to the adoption rate, $\frac{-\Psi'}{\Psi}$, between two consecutive periods depends on the corresponding ratio of the share of renewable energy, $\frac{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}{f+\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}$. To determine the path of i and f, we begin by determining $\widehat{i}$, the long-run i. On a long-run balanced growth path, we have $f = f' = 0$, and $i = i' = \widehat{i}$, where $\widehat{i}$ is determined by

$$\frac{-\Psi'(\widehat{i})}{\Psi(\widehat{i})} = \frac{\beta}{1-\beta} \cdot \xi(1-\alpha). \tag{74}$$

The minimum $\mathcal{E}$ for which f is zero follows from (68):

$$\Psi(\widehat{i})^{\frac{1}{1-\alpha}} \underline{\mathcal{E}} = \frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\tau\pi\left\{\frac{\varphi_L}{1-\beta} + \frac{(1-\varphi_L)\varphi_0}{1-\beta(1-\varphi)}\right\}}. \tag{75}$$

Note that $\underline{\mathcal{E}} < \infty$ only if $\tau > 0$. The representative agent could exhaust the stock of fossil fuel before the productivity of the renewable energy reaches $\underline{\mathcal{E}}$, in which case the price of fossil fuel will be positive. However, once $\mathcal{E} \geq \underline{\mathcal{E}}$, the stock of fossil fuel is not exhausted.

In the period right before the use of fossil fuel ends–i.e., when $f > f' = 0$, following (73) and given that $i' = \widehat{i}$–we have:

$$\begin{aligned} \frac{-\Psi'(i)}{\Psi(i)} &= \beta \cdot \frac{f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}} \left(\xi(1-\alpha) + \frac{-\Psi'(\widehat{i})}{\Psi(\widehat{i})}\right) \\ &= \beta \cdot \frac{f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}} \cdot \frac{\xi(1-\alpha)}{1-\beta}. \end{aligned} \tag{76}$$

Substituting $f' = 0$ in (69) to solve for $f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}$, and noting that $\mathcal{E} = e^{-i}\mathcal{E}'$, the above equation gives

$$\frac{-\Psi'(i)}{\Psi(i)} \cdot \Psi(i)^{\frac{1}{1-\alpha}} \underbrace{e^{-i}\mathcal{E}'}_{\mathcal{E}} = \beta \cdot \frac{1}{\frac{\beta}{\Psi(\widehat{i})^{\frac{1}{1-\alpha}}\mathcal{E}'} + \frac{(1-\beta)}{\frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\tau\pi\left\{\frac{\varphi_L}{1-\beta}+\frac{(1-\varphi_L)\varphi_0}{1-\beta(1-\varphi)}\right\}}}} \cdot \frac{\xi(1-\alpha)}{1-\beta}, \tag{77}$$

which uniquely determines i, given the next period's productivity, $\mathcal{E}'$. Given i and utilizing (69), we have

$$\tilde{f} = \frac{1}{\frac{\beta}{\Psi(\widehat{i})^{\frac{1}{1-\alpha}}\mathcal{E}'} + \frac{(1-\beta)}{\frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\tau\pi\left\{\frac{\varphi_L}{1-\beta}+\frac{(1-\varphi_L)\varphi_0}{1-\beta(1-\varphi)}\right\}}}} - \Psi(i)^{\frac{1}{1-\alpha}} \underbrace{e^{-i}\mathcal{E}'}_{\mathcal{E}}. \tag{78}$$

Note that $\tilde{f}$ is the maximum level of fossil fuel consumption prior to ending its use. That is, if the stock of remaining fossil fuel was larger than $\tilde{f}$, some of the fossil fuel would be left for consumption in the next period.[31] Thus, it is possible that the stock of remaining fossil fuel is in fact lower than $\tilde{f}$. In such cases, equation (69) does not hold, since $f' = 0$ and (68) is an inequality.

[31] Equation (69) holds when both f and f' are positive, but it also holds if $f = \tilde{f}$ and (68) holds with equality for $f' = 0$.

Nevertheless, for any value of $f < \tilde{f}$, we can determine i simply by noting that $\mathcal{E} = e^{-i}\mathcal{E}'$ and using

$$\frac{-\Psi'(i)}{\Psi(i)} \cdot \frac{\Psi(i)^{\frac{1}{1-\alpha}} e^{-i}\mathcal{E}'}{f + \Psi(i)^{\frac{1}{1-\alpha}} e^{-i}\mathcal{E}'} = \beta \cdot \frac{\xi(1-\alpha)}{1-\beta}. \tag{79}$$

When f, $f' > 0$, using (69) to solve for $f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}$ in (73) and noting that $\mathcal{E} = e^{-i}\mathcal{E}'$, we obtain

$$\begin{aligned} \frac{-\Psi'(i)}{\Psi(i)}\Psi(i)^{\frac{1}{1-\alpha}} \underbrace{e^{-i}\mathcal{E}'}_{\mathcal{E}} &= \beta \cdot \frac{1}{\frac{\beta}{f'+\Psi(i')^{\frac{1}{1-\alpha}}\mathcal{E}'} + \frac{(1-\beta)}{\frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\tau\pi\left\{\frac{\varphi_L}{1-\beta}+\frac{(1-\varphi_L)\varphi_0}{1-\beta(1-\varphi)}\right\}}}} \cdot \frac{\Psi(i')^{\frac{1}{1-\alpha}}\mathcal{E}'}{f' + \Psi(i')^{\frac{1}{1-\alpha}}\mathcal{E}'} \\ &\quad \times \left(\xi(1-\alpha) + \frac{-\Psi'(i')}{\Psi(i')}\right), \end{aligned} \tag{80}$$

which allows us to uniquely determine i for a given $\mathcal{E}'$, i', and f'. Then, using i and (69), we obtain the equilibrium path f:

$$f = \frac{1}{\frac{\beta}{f'+\Psi(i')^{\frac{1}{1-\alpha}}\mathcal{E}'} + \frac{(1-\beta)}{\frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\tau\pi\left\{\frac{\varphi_L}{1-\beta}+\frac{(1-\varphi_L)\varphi_0}{1-\beta(1-\varphi)}\right\}}}} - \Psi(i)^{\frac{1}{1-\alpha}} \underbrace{e^{-i}\mathcal{E}'}_{\mathcal{E}}. \tag{81}$$

By using this backward calculation we can determine the entire equilibrium path for all possible initial stock of fossil fuel and renewable productivity levels.[32]

Finally, if $f = 0$, we have

$$V_{\mathcal{E}} = \frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{(1-\beta)(1-\beta\Theta)} \frac{1}{\mathcal{E}}. \tag{82}$$

Hence,

$$\begin{aligned} V\left(k; A, L, \mathcal{E}; 0, \Gamma^p, \Gamma^d\right) &= C + \frac{\Theta}{1-\beta\Theta}\ln k \\ &\quad + \frac{1}{(1-\beta)(1-\beta\Theta)}\left\{\ln A + \theta_l \ln L + (1-\alpha)(1-\theta_k-\theta_\ell)\ln\mathcal{E}\right\} \\ &\quad - \frac{\pi}{(1-\beta)(1-\beta\Theta)}\Gamma^p - \frac{\pi(1-\varphi)}{(1-(1-\varphi)\beta)(1-\beta\Theta)}\Gamma^d, \end{aligned} \tag{83}$$

where C is a constant. Note that log utility, full depreciation, and the structure of the damage function imply that the above expression is linear in Γ^p and Γ^d.

[32] We can show that going backward, i converges to $\widehat{i}^f$ determined by

$$\frac{-\Psi'(\widehat{i}^f)}{\Psi(\widehat{i}^f)} = \frac{\beta e^{\widehat{i}^f}}{1-\beta e^{\widehat{i}^f}}(1-\alpha),$$

where $\widehat{i}^f > \widehat{i}$.

8.5 Productivity Growth in the Fossil Fuel Sector

Here we characterize the equilibrium allocation across the transition in an extension of the model where we allow for growth in the fossil fuel sector. We let $V(k; A, A_f, L, \mathcal{E}, w; \Gamma^p, \Gamma^d)$ denote the value given that the productivity of the fossil fuel sector is A_f and the growth in productivity of the fossil fuel sector is g^f.

The recursive formulation for $V(\cdot)$ is given by

$$\begin{aligned} V\left(k; A, A_f, L, \mathcal{E}, w; \Gamma^p, \Gamma^d\right) \quad = \quad & \max_{i,f} \{\ln((1-\beta\Theta)y) \\ & +\beta V(\beta\Theta y; gA, g^f A_f, g^l l, e^i \mathcal{E}, w-f; \Gamma^{p\prime}, \Gamma^{d\prime})\} \\ & \text{where} \\ & y = e^{-\pi(\Gamma^{p\prime}+\Gamma^{d\prime}-\bar{\Gamma})} A L^{\theta_\ell} \left(A_f f + \Psi(i)^{\frac{1}{1-\alpha}} \mathcal{E}\right)^{(1-\alpha)(1-\theta_k-\theta_\ell)} k^{\Theta}, \\ & \Gamma^{p\prime} = \Gamma^p + \varphi_L f, \\ & \Gamma^{d\prime} = (1-\varphi)\Gamma^d + (1-\varphi_L)\varphi_0 f. \end{aligned} \tag{84}$$

Proceeding as before, we obtain

$$f + \Psi(i)^{\frac{1}{1-\alpha}} \frac{\mathcal{E}}{A_f} \geq \frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\tau\pi\left\{\frac{\varphi_L}{1-\beta} + \frac{(1-\varphi_L)\varphi_0}{1-\beta(1-\varphi)}\right\} + (1-\beta\Theta)V_w}, \tag{85}$$

with equality for $f > 0$. For $f, f' > 0$, using $V_w = \beta V'_{w'}$ and (85), we obtain

$$\frac{1}{f + \Psi(i)^{\frac{1}{1-\alpha}} \frac{\mathcal{E}}{A_f}} \quad = \quad \frac{\beta}{f' + \Psi(i')^{\frac{1}{1-\alpha}} \frac{\mathcal{E}'}{A'_f}} + \frac{1-\beta}{\frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\tau\pi\left\{\frac{\varphi_L}{1-\beta}+\frac{(1-\varphi_L)\varphi_0}{1-\beta(1-\varphi)}\right\}}}. \tag{86}$$

Like before, we use equation (69) to find the equilibrium path of fossil fuel consumption by solving this path backward. The equation determining the optimal choice for i together with the envelope theorem implies

$$\frac{-\Psi'(i)}{\Psi(i)} = \beta \cdot \frac{\frac{\Psi(i')^{\frac{1}{1-\alpha}}\mathcal{E}'}{A'_f f' + \Psi(i')^{\frac{1}{1-\alpha}}\mathcal{E}'}}{\frac{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}{A_f f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}} \left(\xi(1-\alpha) + \frac{-\Psi'(i')}{\Psi(i')}\right). \tag{87}$$

Equation (73) shows how the evolution of the elasticity of the technology adoption cost with respect to the adoption rate, $\frac{-\Psi'}{\Psi}$, in two consecutive periods depends on the corresponding ratio of the share of renewable energy, $\frac{\Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}{A_f f + \Psi(i)^{\frac{1}{1-\alpha}}\mathcal{E}}$.

The minimum $\mathcal{E}$ for which f is zero follows from (85):

$$\Psi(\hat{i})^{\frac{1}{1-\alpha}} \frac{\mathcal{E}}{A_f} = \frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\tau\pi\left\{\frac{\varphi_L}{1-\beta} + \frac{(1-\varphi_L)\varphi_0}{1-\beta(1-\varphi)}\right\}}. \tag{88}$$

Note that with the growth in the productivity of fossil fuel, the level of $\underline{\mathcal{E}}$, for which fossil fuel is no longer used, depends on the productivity of the fossil fuel sector A_f. Therefore, this level exists only if the long-run growth in productivity of the renewable sector, $e^{\widehat{i}}$, which is given by (74), is larger than the growth in productivity of the fossil fuel sector, g^f.[33]

Without loss of generality, we assume that when the productivity in the renewable sector is at $\underline{\mathcal{E}}$, the productivity in the fossil fuel sector is $A_f = 1$. Therefore, for the levels of the renewable productivity $\mathcal{E}$ larger than $\underline{\mathcal{E}}$, when the renewable productivity grows at rate $e^{\widehat{i}}$ while fossil fuel productivity grows at rate g^f, we should have

$$A_f = \exp\left(\log(g^f) \times \left(\log(\mathcal{E}/\underline{\mathcal{E}})/\widehat{i}\right)\right).$$

The rest of the backward solution of the model works similar to the case without productivity growth in the fossil fuel sector. For example, when $f > f' = 0$ we can determine i uniquely from the following expression, which is akin to expression (77):

$$\frac{-\Psi'(i)}{\Psi(i)} \cdot \Psi(i)^{\frac{1}{1-\alpha}} \underbrace{e^{-i}\mathcal{E}'}_{\mathcal{E}} = \beta \cdot \frac{1}{\frac{\beta}{\Psi(\widehat{i})^{\frac{1}{1-\alpha}} \frac{\mathcal{E}'}{A'_f}} + \frac{(1-\beta)}{\frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\tau\pi\left\{\frac{\varphi_L}{1-\beta} + \frac{(1-\varphi_L)\varphi_0}{1-\beta(1-\varphi)}\right\}}}} \cdot \frac{\xi(1-\alpha)}{1-\beta}.$$

And then use i to determine $\tilde{f}$, which is the maximum level of fossil fuel consumption prior to ending its use, from the following expression akin to (78):

$$\underbrace{(g^f)^{-1} A'_f}_{A_f} \cdot \tilde{f} = \frac{1}{\frac{\beta}{\Psi(\widehat{i})^{\frac{1}{1-\alpha}} \frac{\mathcal{E}'}{A'_f}} + \frac{(1-\beta)}{\frac{(1-\alpha)(1-\theta_k-\theta_\ell)}{\tau\pi\left\{\frac{\varphi_L}{1-\beta} + \frac{(1-\varphi_L)\varphi_0}{1-\beta(1-\varphi)}\right\}}}} - \Psi(i)^{\frac{1}{1-\alpha}} \underbrace{e^{-i}\mathcal{E}'}_{\mathcal{E}}.$$

8.6 Calibrating the Spillover Externality

Here we discuss an alternative way of calibrating ξ, the parameter capturing the importance of spillovers in innovation in the renewable energy sector. Although coming from a different viewpoint, this method results in a similar value to the one we use in the paper. In our model, the spillover externality depends on the aggregate amounts of investment and capital stock in the renewable energy sector and affects the firms' productivity. While measuring the direct effect of spillovers on productivity is challenging, one approach is to concentrate on knowledge spillovers. If knowledge, innovation, and productivity improvements are proportional to each other, then quantifying knowledge spillovers can be informative about the magnitude of spillovers in actual productivity improvements. It is common to use patents as a measure of acquired knowledge and

[33] If $g^f \geq e^{\widehat{i}}$, the use of fossil fuel never stops, as it is optimal to leave some fossil fuel for use in the future when the productivity of producing energy with it still exceeds the damage.

patent citations as a measure of the reliance of new discoveries on existing knowledge. Cross-citations can then serve as a measure of connectivity across different technologies, sectors, or geographic locations.

To get a handle on knowledge spillovers related to renewable energy, we investigate the interconnections between new and existing patents in renewable technologies. While we did not have access to cross citations across individual firms, Conti et al (2018) document aggregate cross-citations in renewable technology between the US, the EU, and Japan from 2000 to 2010. These are summarized below:

Citing country	**Domestic**	**Foreign**
EU	.76	.24
Japan	.61	.39
US	.42	.58

For example, assuming ergodicity, this would imply that the average new EU patent citations constitute 76 percent of existing EU patents and 24 percent of existing Japanese and US patents. Simple averaging would lead to a ξ of 0.59, which is quite close to the calibrated value of 0.54 used in the paper.

8.7 Lack of Commitment

In order to explore the implications of commitment, here we briefly consider an extension of the model in the main text. Suppose that the government experiences "electoral death" with probability ω in each period. More precisely, for any period t, with probability $\omega \in (0,1]$, the government learns at the end of t that it will not be around at the beginning of period $t+2$. The implied discounting sequences for the government, β^G, and for the representative agent, β^A, are given by

$$\begin{aligned} \beta^G &= \{1, \beta, \beta^2(1-\omega), \beta^3(1-\omega)^2, ...\} \\ \beta^A &= \{1, \beta, \beta^2, \beta^3, ...\}. \end{aligned} \tag{89}$$

We assume lack of commitment.[34] Thus, at the beginning of each period, the taxes and subsidies are set for the current period. We examine the case where $\xi = 1$, so there are no technology spillovers and the only externality is the one associated with GHG emissions. In each period, the government chooses the Pigouvian tax rate on fossil fuel consumption, τ_t^f. Under no commitment, this tax needs to be set in a time-consistent fashion. The optimal allocation can again be supported by a Pigouvian tax on emissions. This tax is lower than in the case studied in the earlier model.

The government's objective is a modified version of the planner's objective in the previous section and it is given by

[34]See Harstad (2019) for a discussion of time inconsistency in a related model.

$$u(c_0) + \sum_{t=1}^{\infty} (1-\omega)^{t-1} \beta^t u(c_t). \tag{90}$$

All feasibility constraints remain the same. As a result, the optimal allocation is characterized by similar FOCs as in the previous section, with the only modification being in the discount sequence. We can demonstrate the following.

Proposition 4 *(1) The government's optimal allocation can be supported by a Pigouvian tax given by* $\tau_t^f = \pi_t y_t^*(1-d_0) + \sum_{j=1}^{\infty}(1-\omega)^{j-1}\beta^j \frac{u'(c_{t+j}^*)}{u'(c_t^*)}\pi_{t+j} y_{t+j}^*(1-d_j)$*, where* $\{c_t^*, y_t^*, i_t^*\}_{t=0}^{\infty}$ *is the solution to the government's problem, and* $1-d_j = \varphi_L + (1-\varphi_L)\varphi_0(1-\varphi)^j$*. (2) If* $u(c) = \log(c)$*,* $\alpha_r = \alpha_f = \alpha$*,* $\pi_t = \pi$*, all* t*, and* $\delta = 1$*, then* $\tau_t^f = y_t \pi \left[\varphi_L + (1-\varphi_L)\varphi_0 + \frac{\beta\varphi_L}{1-(1-\omega)\beta} + \frac{\beta(1-\varphi)(1-\varphi_L)\varphi_0}{1-(1-\omega)(1-\varphi)\beta}\right]$ *does not depend on the growth rate of the economy. (3) The tax is strictly lower than the optimal Pigouvian tax of the previous section for all* t*.*

8.8 A Continuous Time Interpretation

Here we provide a continuous time foundation of how our modeling of the technology adoption cost in the main text can be interpreted as the costs associated with the scraping process. The instantaneous production function in this case is given by

$$e_j^r(t) = [\mathcal{E}_j(t)]^{1-\alpha_r} \left[k_j^r(t)\right]^{\alpha_r}. \tag{91}$$

As the new technology adoption rate, $i_j(t)$, requires scrapping of current capital, capital evolves according to

$$\frac{dk_j^r(t)}{k_j^r(t)} = -f(i_j(t))dt, \tag{92}$$

where $f' > 0$. Adoption of a newer technology, $i_j(t)$, improves the productivity of firm j according to

$$\frac{d\mathcal{E}_j(t)}{\mathcal{E}_j(t)} = g(i_j(t))dt, \tag{93}$$

where $g' > 0$. The resulting change in production is

$$de_j^r(t) = \alpha_r [\mathcal{E}_j(t)]^{1-\alpha_r} \left[k_j^r(t)\right]^{\alpha_r - 1} dk_j^r(t) + (1-\alpha_r) [\mathcal{E}_j(t)]^{-\alpha_r} \left[k_j^r(t)\right]^{\alpha_r} d\mathcal{E}_j(t). \tag{94}$$

After replacing for the $\mathcal{E}_j(t)$ and $k_j^r(t)$ processes, we obtain

$$de_j^r(t) = \Psi[i_j(t)] [\mathcal{E}_j(t)]^{1-\alpha_r} \left[k_j^r(t)\right]^{\alpha_r} dt, \tag{95}$$

where $\Psi[i_j(t)] \equiv (1-\alpha_r)g(i_j(t)) - \alpha_r f(i_j(t))$. We assume that $\Psi' < 0$.

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