Demographics and the U.S. Economy

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Abstract

How much can demographic changes account for trends in the U.S. economy? This paper shows that a heterogeneous-agent, overlapping-generations model with historical demographic flows can generate several features of the U.S. economy over the past several decades, including a secular decline in economic growth, a rise in savings relative to GDP, a corresponding decline in real interest rates, and, in part, changes in wealth inequality. Simulations show that education and immigration contributed to these trends, but the main driver was an aging of the U.S. population driven by declining birth rates and increased life expectancy. Counterfactual analysis shows that increased immigration can improve near-term economic growth (total and per capita) and offset some aging of the population. However, there is little that realistic policy intervention can do to offset these trends in the long-run.

Keywords: overlapping generations; demographics; economic policy.
1 Introduction

Since the 1980’s, the U.S. economy has experienced several secular trends: declining growth rates of total GDP, GDP per capita, investment, and labor hours; a rising capital-to-GDP ratio; declining interest rates; and rising wealth and income inequality. This paper presents a macroeconomic model with a rich demographic structure that explains these trends with plausible corresponding magnitudes. Moreover, the model includes immigration and educational attainment to discern how much each of these factors contributes to the trends. Finally this paper evaluates alternative immigration and fiscal policies to determine how these recent trends could be impacted by the decisions of policymakers.

The model is a dynamic general equilibrium economy with overlapping generations of individuals. To construct realistic demographics, the framework accounts for varying birth rates, mortality rates, and educational attainment by cohort, as well as immigration flows by age, year, and educational attainment. A simplified stationary tax structure is also included to evaluate the contribution of taxation to the observed trends and to evaluate the effects of policy reforms.

The baseline model generates several features of U.S. demographics and the U.S. economy since around 1970. To understand how much education and immigration each contribute to the observed trends, the baseline economy is solved without each. The results suggest that both are significant factors. What remains is the role of age demographics and, in particular, an aging of the population, which accounts for a considerable share of the observed trends.

Since the framework is well-suited for policy analysis, three reforms are considered. Each policy is extreme by design for the purpose of illustration. In the first reform, the immigration rate is increased fourfold. The results of this reform are generally positive, improving economic growth (total and per capita) and increasing the share of the working-age population. In the second reform, skill-based immigration policy becomes maximally restrictive, as the share of college-attaining immigrants is set to 100%. This reform also improves economic growth and modestly increases tax revenue. Finally, the third reform is a 20% tax cut. The results show that this aggressive tax cut has a limited effect on economic growth, but over longer time horizons, GDP improves roughly
0.5% for every percentage point reduction in tax-revenue-to-GDP.

The prospect of declining population growth causing a secular decline in macroeconomic growth in the 20th century was recognized at least as early as Hansen (1939). At that point in time, it would have been difficult to anticipate the impending population resurgence of the mid 20th century. The baby boom of that era led some economists to question the persistence of declining fertility rates and, in some cases, to question the direction of causality between population and economic growth. Kuznets (1967), for example, cites: “Indeed, the question that we wish to examine is that emphasized in the widespread discussion of the current population ‘explosion.’ To what extent does a high rate of population growth impede the growth of per capita product.” This paper supports the converse by showing that exogenous demographic changes can generate plausible historical economic growth patterns—particularly in the years since 1970.

As the fertility rates once again plummeted in the 1960’s and life expectancy surged, the literature again turned to implications of an aging of the population. McMillan and Baesel (1990), for example, estimate an inverse relationship between the age of the population and the real interest rate—a finding supported by life-cycle models. The authors also correctly projected a decline in the real interest rate as the baby boom generation neared retirement age. Much of the literature also evaluated the consequences of an aging population for economic policy, including De Nardi, Imrohoroglu, and Sargent (1999), Elmendorf and Sheiner (2000), and Nishiyama (2015). Storesletten (2000) contributes to this literature, as well, by considering the fiscal effects of immigration in an aging population. This paper complements those findings by measuring the fiscal consequences of alternative immigration reforms.

Persistent macroeconomic trends in the U.S. since the 1980’s and, in particular, declining economic growth and real interest rates, have been studied in the literature and are often referenced as Secular Stagnation. This phenomenon and alternative hypotheses have been summarized in Summers (2014), Eichengreen (2015), Gordon (2015), and Summers (2015). This paper shows that nearly all of the trends cited in the literature can be explained by a life-cycle model with rich demographic structure. In particular, the model shows that an aging population leads to an increase in the share of individuals near the peak of lifetime wealth, generating an increase in capital, relative
to output—a so-called capital deepening. This happens despite declining investment growth, causing a decline in the real interest rate. To the extent that these trends are generated by demographic changes within a life-cycle model, this paper confirms several findings, including those in Carvalho, Ferrero, and Nechio (2016) and Gagnon, Johannsen, and Lopez-Salido (2016). The model also accounts generates very long-term features of the evolution of wealth inequality, though it can not generate the observed dip over the last four decades. This conclusion provides evidence against the role of demographic changes in causing that transitory decline, though the mechanism should be explored in greater detail.

The rest of the paper is organized as follows: Section 2 presents the theoretical model, and Section 3 describes the calibration. Section 4 shows the baseline results, and Section 5 inspects the underlying mechanisms. Section 6 evaluates policy reforms, and Section 7 concludes.

2 Model

The model is a heterogeneous-agent dynamic general equilibrium economy with overlapping generations of agents. Following Bewely-Aiyagari-Huggett models, individuals face uninsurable idiosyncratic productivity risk, resulting in market incompleteness. Individuals take prices and policy as given and choose quantities to maximize lifetime utility, while firms take prices and policy as given to maximize profits. Prices adjust to clear the goods, asset, and labor markets.

2.1 Individual Optimization

In every period, individuals incur a persistent idiosyncratic labor productivity shock, $\varepsilon \in \{\varepsilon_1, \ldots, \varepsilon_N\} = \mathcal{E}$. The stochastic process for the productivity shock is characterized by a discrete-time Markov chain with corresponding transition probabilities that remain constant for every age and time period. Productivity shocks at the beginning of the working life are drawn from distribution $\mathcal{N}(0, \sigma_{\text{initial}}^2)$.

The decision of an individual born in period $j$, i.e., a cohort-$j$ individual, with education level
\( e \in \{ h, l \} \) is characterized by the following Bellman equation:

\[
V^e_{j,t}(a_{j,t}, \varepsilon) = \max_{c_{j,t}, n_{j,t}, a_{j,t+1}} \left\{ \frac{(c_{j,t}^e \gamma (1 - n_{j,t}^e)^{1-\gamma})^{1-\sigma}}{1-\sigma} + s_{j,t} \beta E_{\varepsilon'} S_{j,t+1} \right\} V^e_{j,t+1}(a_{j,t+1}, \varepsilon') \tag{1}
\]

s.t. \( c_{j,t}^e = w_t \varepsilon \epsilon^e_t z_{t-j+1} n_{j,t}^e + (1 + r_t) a_{j,t}^e - a_{j,t+1}^e - \phi_e(\cdot), \tag{2} \)

and \( \phi_e(\cdot) = \tau_e \left( w_t \varepsilon \epsilon^e_t z_{t-j+1} n_{j,t}^e + r_t a_{j,t}^e \right), \tag{3} \)

where the dynamics of productivity shocks are represented using recursive notation. Variable \( t \) denotes the time period, so that the individual’s age is \( t - j + 1. \) The individual survives to the next age with cohort- and age-dependent survival probability \( s_{j,t} \) and lives to a maximum of age \( J. \) In every living year, the individual takes prices \( \{ w_t, r_t \} \), time-dependent education premium \( \epsilon^e_t \), deterministic age-dependent labor productivity \( z_{t-j+1} \), and income tax function \( \phi_e(\cdot) \) as given and chooses over consumption \( c_{j,t}^e \), labor supply \( n_{j,t}^e \), and assets \( a_{j,t+1}^e \). The individual gains utility from consumption \( c_{j,t}^e \) and leisure \( 1 - n_{j,t}^e. \) The latter reflects the time normalization to unity and the mutually exhaustive allocation of time between labor and leisure. Finally, borrowing is restricted in every period, and agents discount future utility at rate \( \beta. \)

The income tax function \( \phi_e(\cdot) \) is linear and depends only on education type. This allows for variation in the marginal tax rate without requiring the calibration of a progressive tax function. More importantly, this specification creates tax variation across education types, which helps to understand the impact of skill-based immigration policies on the government budget.

### 2.2 Demographics

The economy is populated by natives and immigrants. For education level \( e \), cohort \( j \), natives in period \( t \) are represented by measure \( \mu^e_{j,t} \), while immigrants have measure \( \tilde{\mu}^e_{j,t} \). In the next period, the measure of natives and immigrants represented by this cohort will be, respectively:

\[
\mu^e_{j,t+1} = s_{j,t} \mu^e_{j,t} \quad \text{(natives)} \tag{4}
\]

\( \tilde{\mu}^e_{j,t+1} = s_{j,t} \tilde{\mu}^e_{j,t} \quad \text{(immigrants)} \tag{5}
\]

1The general notation for a variable \( x \) with two subscripts is \( x_{\text{cohort,} \text{time}} \). All other variables’ subscripts are explained in context.
\[
\tilde{\mu}_{j,t+1}^e = s_{j,t}\mu_{j,t}^e + m_{m,t+1}^e \quad \text{(immigrants)},
\]

where \(m_{m,t+1}^e\) is the net inflow of immigrants corresponding to the specific type. By assumption, immigrants enter into the economy with the same capital as natives of the same age and education.

In any period \(t\), the total population \(M_t\) is determined as follows:

\[
M_t = \sum_{j=t}^{t-J+1} \sum_{e \in \{h,l\}} (\mu_{j,t}^e + \tilde{\mu}_{j,t}^e).
\]

The inflow of native cohorts is determined by the birth rate, which is defined as the measure of newborn entrants as a fraction of population the period before.\(^2\) Accordingly, constructing the period \(t+1\) population starts with calculating age 1 individuals as a fraction of the period \(t\) population. For any time-dependent birth rate \(\zeta_t\) the age 1 individuals in period \(t+1\) are calculated as follows:

\[
\sum_{e \in \{h,l\}} \mu_{t+1,t+1}^e = \zeta_t M_t.
\]

Finally, education states are permanent and determined at birth. Native shares of each education type for a cohort \(t+1\) are determined by \(\lambda_t^e\), where:

\[
\mu_{t+1,t+1}^e = \zeta_t \lambda_t^e M_t.
\]

An immigrant in the model is defined as an individual that enters the population after birth. The share of immigrants entering the population in any period \(t\) is determined by the immigration rate \(\psi_t\), which is relative to the population \(M_t\). In any period \(t\), the variable \(\tilde{\lambda}_{j,t}^e\) determines the education share and age (cohort) share of immigrant inflows.\(^3\) Then, the measure of immigrants of

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\(^2\)This conflicts slightly with the OECD definition, which calculates the birth rate as a share of the population in the same period. However, the model's assumption simplifies the computation and provides a reasonable estimate for the discrete approximation of a continuous process.

\(^3\)In particular, \(\tilde{\lambda}_{j,t}^e\) is an element of the joint discrete probability space over education and age, so that \(\sum_{j=t}^{t-J+1} \sum_{e \in \{h,l\}} \lambda_{j,t}^e = 1\).
cohort $j$ and education $e$ flowing into period $t + 1$ is:

$$m_{j,t+1}^e = \psi_t \tilde{\lambda}_{j,t}^e M_t. \quad (9)$$

Then, for any initial age distribution \( \{\mu_{j,t}^e, \tilde{\mu}_{j,t}^e\}_{j=t,J+1} \), Equations 4-9 jointly characterize the transition function.

If the population growth rate is positive in the limit, then the population sequence is divergent. Similarly, if the population growth rate is negative in the limit, then the population converges to zero. Along the transition, however, stable growth can be achieved in the form of relative age convergence. Let the relative population be defined by the vector:

$$\Phi_t = \left\{ \sum_{e \in \{h,l\}} \left( \mu_{j,t}^e + \tilde{\mu}_{j,t}^e \right) \right\}_{j=t}^{t-J+1} \cdot \left( M_t \right) \quad (10)$$

The vector $\Phi_t$ is a normalization resulting in the discrete probability distribution over ages. If the relative population vectors \( \{\Phi_t, \Phi_{t+1}, \ldots\} \) is Cauchy sequence, then there exists a stationary distribution over ages, $\Phi^*$, such that $\lim_{t \to \infty} \Phi_t = \Phi^*$, a stable population. Stable populations will provide initial and terminal demographic steady states corresponding to the time paths of interest.\(^4\)

If the government can control the immigration rate $\psi_t$ and probability distribution over age and education $\tilde{\lambda}_{j,t}^e$, then immigration policy can augment the population by any magnitude with respect to age and education. This specification allows for analysis of skill-based immigration policies, but it could also allow for policies that target specific age groups. An ongoing assumption is that any arbitrary measure of immigrants is willing to enter the economy at any point in time.

### 2.3 Firms

In every period, perfectly competitive firms borrow capital $K_t$ and efficient labor $L_t$ to produce output $Y_t$ in a perfectly competitive market. Output is produced using constant-return-to-scale

\(^4\)For a discussion on stable populations as they relate to age and immigration, see Weil et al. (1997).
(CRS) technology:

\[ Y_t = F(K_t, A_t L_t), \]  

where \( A_t \) is labor-augmenting technological improvement that grows at a constant rate \( g \), so that \( A_{t+1} = (1 + g)A_t \) for all \( t \). In the process of production, capital depreciates at rate \( \delta \). Finally, firms pay individuals real interest rate \( r_t \) to borrow each unit of capital and pay real wages \( w_t \) for each unit of effective labor.

The problem of the firm can be written as:

\[
\max_{K_t, L_t} K_t^\alpha (A_t L_t)^{1-\alpha} - (r_t + \delta)K_t - w_t L_t,
\]  

where \( \alpha \) is the capital share of production. Optimal conditions of the firm imply the following equations:

\[
r_t = \alpha \left( \frac{K_t}{A_t L_t} \right)^{\alpha-1} - \delta, \]  

\[
w_t = (1 - \alpha) \left( \frac{K_t}{A_t L_t} \right)^\alpha. \]  

Finally, perfect competition and CRS technology ensure that profits are zero in equilibrium.

2.4 Government

The government finances an exogenous stream of expenditures, \( G_t \). Let \( R_t \) denote aggregate net government receipts as follows:

\[
R_t = \sum_{j=t}^{t+1} \sum_{e \in \{h,l\}} \left( \mu_{j,t}^e + \tilde{\mu}_{j,t}^e \right) \phi_e(\cdot). \]  

Then, the budget constraint of the government is denoted:

\[
G_t = R_t + B_t. \]  

The variable \( B_t \) in Equation 16 is accidental bequests, which are collected as government receipts.
2.5 Equilibrium

Define $\psi_{e,j,t}^e$ as a probability measure on subsets of the state space for every education level, $e$, age $j$, and time $t$. Let $(\mathcal{X}, B(\mathcal{X}), \psi_{e,j,t}^e)$ be a probability space, where $\mathcal{X} = [0, \infty) \times \mathcal{E}$ and $B(\mathcal{X})$ is the Borel $\sigma$-algebra on $\mathcal{X}$. For every set $B \in B(\mathcal{X})$, $\psi_{e,j,t}^e(B)$ is the fraction of age $j$ agents at time $t$ whose individual states lie in $B$ as a portion of all age $j$ agents with education $e$ at time $t$.

Let $x \in \mathcal{X}$ denote an individual’s state. A dynamic general equilibrium is defined as a set of prices $\{w_t, r_t\}$, quantities $\{c_{e,j,t}^e(x), n_{e,j,t}^e(x), a_{e,j,t+1}^e(x)\}$ such that:

1. Given prices and government policy, individuals’ choices satisfy Equation 1 - Equation 3,
2. Prices are determined in competitive markets according to Equation 13 and Equation 14,
3. Markets clear:
   (a) $Y_t = C_t + K_{t+1} - (1 - \delta)K_t + G_t$
   (b) $K_t = \sum_{j=t}^{t-J+1} \sum_{e \in \{h,l\}} \left( \mu_{e,j,t}^e + \tilde{\mu}_{e,j,t}^e \right) \int_{\mathcal{X}} a_{e,j,t+1}^e(x) d\psi_{e,j,t}^e$
   (c) $L_t = \sum_{j=t}^{t-J+1} \sum_{e \in \{h,l\}} \left( \mu_{e,j,t}^e + \tilde{\mu}_{e,j,t}^e \right) \int_{\mathcal{X}} e_{z_{t-j} + 1} n_{e,j,t}^e(x) d\psi_{e,j,t}^e$
4. Government budget constraint (16) is satisfied.
5. Accidental bequests received by the government are determined according to

   $B_t = \sum_{j=t}^{t-J+1} \sum_{e \in \{h,l\}} \left( \mu_{e,j,t}^e + \tilde{\mu}_{e,j,t}^e \right) \int_{\mathcal{X}} (1 - s_{j,t}) a_{e,j,t+1}^e(x) d\psi_{e,j,t}^e. \quad (17)$

3 Calibration

This section describes the determination of model parameters. This includes several deep parameters that are consistent with values in the macroeconomics literature and others that are calibrated to match the data. The remaining parameters are estimated from the data and described below.

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5This equilibrium definition follows Huggett [1996]
Table 1: Scalar parameters, symbols, and values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient of Relative Risk Aversion</td>
<td>$\sigma$</td>
<td>3</td>
</tr>
<tr>
<td>Consumption Share of Utility</td>
<td>$\gamma$</td>
<td>0.35</td>
</tr>
<tr>
<td>Discount Factor</td>
<td>$\beta$</td>
<td>1.025</td>
</tr>
<tr>
<td>Maximum Age</td>
<td>$J$</td>
<td>120</td>
</tr>
<tr>
<td>Capital Share</td>
<td>$\alpha$</td>
<td>0.36</td>
</tr>
<tr>
<td>Depreciation Rate</td>
<td>$\delta$</td>
<td>0.085</td>
</tr>
<tr>
<td>Labor Productivity Growth Rate</td>
<td>$g$</td>
<td>0.015</td>
</tr>
<tr>
<td>High Marginal Tax Rate</td>
<td>$\tau_h$</td>
<td>0.1208</td>
</tr>
<tr>
<td>Low Marginal Tax Rate</td>
<td>$\tau_l$</td>
<td>0.0615</td>
</tr>
</tbody>
</table>

Table 1 summarizes several scalar parameters of the model. Discount factors are often chosen to generate a capital-to-labor ratio around 3. The baseline model herein delivers a capital-to-output ratio that takes a range of values from around 2.6% in 1950 to 3.2% in 2019. The parameters $\sigma$ and $\gamma$ imply coefficient of relative risk aversion in consumption of 2.3, which gives an elasticity of intertemporal substitution of 0.43. This falls within the range of values reported in Havranek et al. (2015).

Model ages and real-world ages are the same. Individuals in the model are assumed to begin their economic lives at age 20. Before then, the individual makes no decision, and all corresponding values, with the exception of population weights, are zero. In addition to the scalar parameters in Table 1, the vector of labor productivity for all ages $i$ is determined according to $z_i = 0.4817 + 0.0679(i - 21) - 0.0013(i - 21)^2$, which Gervais (2012) provides as a smoothed version of estimates found in Hansen (1993).

Marginal tax rates are estimated from 2017 Current Population Survey (accessed via IPUMS6). The estimates correspond to the average marginal tax rate of each group, as defined by college attainment. This implementation of income taxation in the model creates a type of progressivity in personal income taxes without requiring the calibration of a progressive tax function. Baseline tax revenue varies from about 5.5% of GDP in 1950 to around 6.5% of GDP in 2019. This yields

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somewhat lower personal income tax revenue relative to historical data values, which have varied
between 5.7% and 9.9% of GDP over the same time period.\\footnote{https://www.taxpolicycenter.org/statistics/source-revenue-share-gdp}

Figure 1: Historical Rates, Projections, and Extrapolations

Historical birth rates, $\zeta_t$, shown in Figure 1(a) are derived from the National Center for Health
Statistics and span a time period of 1909 to 2015. Projected birth rates are provided by the U.S.
Census Bureau in decennial frequency from 2020 to 2060. Values for the years in between are
interpolated, and values past 2060 are linearly extrapolated up to year 2095, which is the last
cohort year that survival probabilities projections are provided.

Historical and projected survival probabilities are provided by the Social Security Administration
for all ages 0 to 120 and time period spanning the lifetimes of cohorts born between 1900 and 2095. Separate survival probabilities are provided for males and females. Since individuals in the model are gender-neutral, the survival probabilities from the data are equally-weighted means of each gender for a given cohort and age.

Birth rates and survival probabilities are both noisy processes that create difficulty for computation of the equilibrium price vector. To relieve this burden, each series is smoothed by a low-level Hodrick-Prescott (HP) filter. This process results in negligible changes to the baseline results but helps to compute equilibrium prices with greater precision.

The immigration rate, $\zeta_t$, shown in Figure 1(b) is calculated as a percentage inflow, relative to the population in the specific year. For most of the time span in which immigration data was collected, these values were measured on a decennial basis. To generate annual immigration rates, total immigrant inflows were linearly interpolated for every year, and a percentage increase, relative to the previous year’s population was calculated. The immigration projection is provided by the U.S. Census Bureau up till the year 2060. Subsequent values are held constant at the projection for 2060.

Education types are determined by the college attainment rate between the ages of 25 and 34. For natives, college attainment rates are calculated from data provided in the U.S. Census Bureau educational attainment tables, as shown in Figure 1(c). To account for ongoing variations in the college attainment rates, the values are linearly extrapolated for the decade following the final observation in 2019. For values before the first observation in 1940, values are assumed to be constant at the 1940 value. As with other series, college attainment rates are smoothed with a low-level HP filter.

For immigrants, the values are estimated from American Community Survey data, acquired through IPUMS. The same definition of college attainment is applied. Although historical immigrant college attainment rates have been lower than native college attainment rates, the gap between the two values has recently declined. In particular, college attainment rates are improving

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faster for immigrants than they are for natives, as shown in Figure 1(d). Then, using the same extrapolation rule that was used for natives, the projection suggests that immigrants will eventually surpass natives in educational attainment.

Figure 2: Immigrant age distribution by college attainment, 1970 and 2017.

In addition to college attainment rates, the age distribution of entering immigrants in every year is needed for the variable $\tilde{\lambda}_{j,t}$. To derive this variable, the age distribution of immigrant inflows by college attainment is estimated in every available year, and unavailable years are interpolated. The resulting distributions are shown in Figure 2 for the years 1970 and 2017. The age distribution for college-attaining immigrants is censored at 18 years of age.

Some share of immigrants that arrive without college attainment still gain college attainment. To account for this, in every year, the percentage of aged 25-34 who entered before the age of 18 is calculated and applied to generate the share of immigrant children who reach college attainment, as shown in Figure 1(e). Since the available data only began in 1970, the initial value largely

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This could happen as a result of policy or selection of immigrants (or both). However, projections do not need to take a position on the cause, only that the status quo is maintained.
overstates the college attainment of immigrants in earlier decades. To adjust for this, the values are linearly extrapolated into the past, capturing the trajectory of changes in college attainment rates, and making earlier values more consistent with other estimates of immigrant college attainment rates from that time period. Finally, as with other series, the values are linearly extrapolated into the future for ten years and the entire series is smoothed with a low-level HP filter.

4 Baseline Results

4.1 Computation

To generate the baseline economy, a stable population, $\Phi^*_{\text{initial}}$, corresponding to the earliest available data is approximated. Then, the variables change over the data horizon, and the population in every year is determined according to the demographic transition equations. Finally, the variables are held constant at the values corresponding to the last available set of projections, and a terminal stable population, $\Phi^*_{\text{terminal}}$, is approximated.

To generate a set of equilibrium values over the transition path, and initial steady-state equilibrium corresponding to the initial demographic distribution is solved. To avoid a demographic shock to the initial steady-state economy, additional periods are added to the beginning of the transition path until the initial demographic change is far enough into the future that early cohorts have a negligible reaction to it. Fifty initial periods is enough to satisfy this condition. Then, the transition path of the economy is solved until $\Phi^*_{\text{terminal}}$ is reached. By that particular period, the economy has also reached a terminal steady state to a high degree of precision. The entire path is approximately 600 periods.

4.2 Demographics

This section compares the model’s simulated demographics with the corresponding data values. In particular, the series represent age structure, immigrant share of the population, and shares of

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10The earliest available data and the furthest projection are both survival probabilities, which span cohorts born in every year from 1900 to 2095.
11Since the earliest available data is for 1900, this implies an initial year of the transition path of 1850.
the population with college attainment.

![Graph](image1.png)

(a) 65+ to 15-64 population ratio.

![Graph](image2.png)

(b) Immigrant share of population

![Graph](image3.png)

(c) College attainment share of population

![Graph](image4.png)

(d) College attainment share of 25-34 population

Figure 3: Baseline demographics, model vs. data

Since many of the macroeconomic trends are driven by changes in the age structure of the population, such values in the model are the most important to match to the data. Figure 3(a) shows the retiree dependency ratio, which is defined as the ratio of the population above the age of 65 to the population ages 15-64. The simulated values provide a close match to the data, perhaps only exceeding the data by a small amount in the latter years. The sharp rise in the dependency ratio can be explained by a combination of declining fertility rates and rising life expectancy over the last several decades. The sharp increase in the final decade reflects the aging of the baby boom generation.

Immigration data shows a heightened share of immigrants in the early 20th century. The share then declined persistently until the mid-1960’s before rising back up. This trend likely reflects immigration restrictions implemented by the Immigration Act of 1924,\(^\text{12}\) which were subsequently reversed by the Immigration and Nationality Act of 1965.\(^\text{13}\)

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\(^{12}\)https://history.state.gov/milestones/1921-1936/immigration-act

With the raw immigration data, the model understates the immigrant share of the population in every year by roughly 3 percentage points. By scaling the immigration rate up by one person per thousand, or roughly 25%, the simulated immigration share provides a much closer match to the data, as shown in Figure 3(b).

For most of the time series, the share of the population with college attainment in the simulated data provides a close match to the data. Figure 3(c) shows, however, that after 1970, the model understates the data by an increasing amount. This likely reflects a growing share of individuals over 35 years of age that reach college attainment later in life. This is supported by Figure 3(d), which shows that for the 25-34 age group, the college attainment rate in the simulation provides a close match to the data.

4.3 Economy

This section compares the baseline economy to the corresponding data, while inspection of the mechanism is reserved for the next section. Each graph in Figure 4 presents the model’s baseline economy and the corresponding values from the data. The data values show the output of a HP filter given a low and a high smoothing parameter (\(\lambda\)). Since values from the data have an annual frequency, the low smoothing parameter is set to \(\lambda = 6.25\), as suggested in Ravn and Uhlig (2002). A high smoothing parameter value of \(\lambda = 1000\) is also supplied to highlight smoother, longer-term trends.

Consider first the growth rate of real gross domestic product (GDP) shown in Figure 4(a). For GDP growth, as well as most variables, the model provides a close match to the data in the years since 1980. In the two decades preceding 1970, the model understates the data. This finding is consistent with Gagnon, Johannsen, and Lopez-Salido (2016), which also accounts for macroeconomic trends through demographic changes. In the three decades from 1970-2000, U.S. GDP remained relatively stable around 3.25% annual growth. Then in the two decades following 2000, GDP growth declined steadily to around 2.5%. This model shows a similar period of stability.

\[\text{U.S. Bureau of Economic Analysis, Real Gross Domestic Product (GDPCA), retrieved from FRED, Federal Reserve Bank of St. Louis}\]
in the last three decades of the twentieth century, followed by a sharper decline thereafter. The large dip in the data in that period correspond to the severity of the Great Recession, but the average annual GDP growth rate in the subsequent decade (2009-2019) is 2.3%.
Since the model understates the level of economic growth in the first two decades, the same is true for model values of GDP per capita, as shown in Figure 4(b). Since 1970, however, both the model and the data show a decline from around 2.5% in 1970 to below 2% in 2019. Nearly half of that decline happens just in the final decade, reflecting a similar pattern as total GDP growth.

Investment growth is a volatile data series, but the model provides a close match to the data in the years since 1970, as shown in Figure 4(c). As with GDP, the model understates investment growth in the two decades from 1950-1970, likely contributing to the disparity. Both the model and the data show a secular decline in investment growth starting around 1990, which is the subject of several research papers studying secular stagnation. This finding supports demographics as the cause of declining investment growth.

Productive labor is a model input to production which has no clear real-world analog. Model labor hours, however, can be compared with its real-world analog, as in Figure 4(d). As with the other components of output, model growth in labor hours understates the data in the two decades spanning 1950-1970, possibly also explaining the disparity. In the time thereafter, the model generates a close match to the data. Both the model and the data show a secular decline in the annual growth rate of labor hours starting in 1970 from around 2% to around 1% in 2019.

Consider next the capital-to-output ratio. In standard neoclassical growth models, individuals’ savings equals firms’ capital stocks. In reality, firms’ capital and individuals’ savings may differ for several reasons, including international finance and public debt markets. Accordingly, the model values are compared to a measure of the capital stock and a measure of personal wealth. Consider first the capital-to-output ratio in Figure 4(e), where capital in the data is measured by the stock of U.S. capital. The model generates a persistent increase in the capital-to-output ratio, rising from around 2.6 in 1950 to 3.25 in 2019. Although the series shows long-term volatility, it trends

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15 U.S. Bureau of Economic Analysis, Real gross domestic product per capita [A939RX0Q048SBEA], retrieved from FRED, Federal Reserve Bank of St. Louis
16 U.S. Bureau of Economic Analysis, Real Gross Private Domestic Investment [GPDIC1], retrieved from FRED, Federal Reserve Bank of St. Louis
17 U.S. Bureau of Economic Analysis, Hours worked by full-time and part-time employees [B4701C0A222NBEA], retrieved from FRED, Federal Reserve Bank of St. Louis
18 U.S. Bureau of Economic Analysis, Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods [KIWTOTLIES000], retrieved from FRED, Federal Reserve Bank of St. Louis
19 U.S. Bureau of Economic Analysis, Gross Domestic Product [GDP], retrieved from FRED, Federal Reserve Bank of St. Louis
modestly upward over time but overstates the model values in the earlier periods. For analysis of personal wealth, Figure 4(f) shows U.S. net worth as a share of GDP, which is scaled down for comparison. Although the model values differ from the data values, the model provides a reasonable match for the pattern in the data. Moreover, real values may account for valuations in ways that this model does not (e.g., stocks and real estate valuations).

Figure 4(g) compares model interest rates to real interest rate. The real interest rate is calculated as the 10-year U.S. Treasury rate net of the Consumer Price Index. Again, because of the variability in the definition of a real-world interest rate, the data values are scaled to the model’s range for emphasis. The model shows an initial decline until around 1970. Then, the values increase slightly until the early 1980’s before declining again steadily over the next 40 years. The data shows a similar pattern, with interest rates declining sharply after 1980.

4.4 Wealth and Income Distributions

The U.S. has experienced secular trends in wealth and income inequality, as measured by the respective Gini coefficients and shares of each held by subsets of the population. In particular, both wealth and income inequality have increased relative to their respective values in 1950. Understanding what role demographics might play in these secular trends is central to determining the role that policy might play in addressing inequality.

Figure 5 and Figure 6 show how the model compares to historical data. Although the current version of the Survey of Consumer Finances (SCF), which is commonly used to measure wealth inequality, contains data only back to 1989, Kuhn, Schularick, and Steins (2020) provides expanded estimates of the SCF variables dating back to 1949. These estimates provide a basis of comparison

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20Board of Governors of the Federal Reserve System (US), Households and Nonprofit Organizations; Net Worth, Level [TNWBSSHNO], retrieved from FRED, Federal Reserve Bank of St. Louis
21Using an alternative measure of capital-to-output, Gourio and Klier (2015) also shows a steady increase in the capital-to-output ratio, though the rise starts earlier, around 1970, following a flat period in the two preceding decades.
22Board of Governors of the Federal Reserve System (US), 10-Year Treasury Constant Maturity Rate [GS10], retrieved from FRED, Federal Reserve Bank of St. Louis
23U.S. Bureau of Labor Statistics, Consumer Price Index for All Urban Consumers: All Items in U.S. City Average [CPIAUCSL], retrieved from FRED, Federal Reserve Bank of St. Louis
24For robustness, Moody’s Seasoned Baa Corporate Bond Yield was used in the place of the 10 year Treasury. This generally resulted in a similar pattern at higher interest rates.
for the model values.

Properties of the productivity shock process were chosen to match the income and wealth distribution in 2020. Figure 5 shows how the model can reasonably match wealth inequality at the endpoints of the specified time period. However, the model cannot explain the relative dip in wealth inequality in the interim periods. The model is also unable to match trends in income inequality over the specified time period, indicating that other factors likely played a role.

Figure 5: Wealth distribution in the baseline economy.

4.5 Macroeconomic Projections

Figure 7 shows the model’s baseline projections, given various assumptions and projections of demographics, educational attainment, and immigration. In the long run, the economy reaches a steady state corresponding to terminal values of all model inputs. As the economy tends towards this steady state, trends along the transition are driven by improvements in educational attainment, declining population growth, and a continued aging of the population.

In the terminal steady state, economic growth is determined by the population growth rate
and the fixed exogenous growth rate of labor productivity \((g)\). In the terminal steady state, the population growth rate is approximately 0.2\%, so the long-run GDP growth rate is about 1.7\%. In the initial decade of the projection, however, the educational attainment rate continues rising before stabilizing at a heightened value. Population growth and educational attainment have opposite effects on economic growth, so economic growth only declines slightly until about 2035. Around that time, educational attainment stabilizes and population growth continues decline. This causes an accelerated decline thereafter.

The growth rate of GDP per capita reverses an earlier decline leading up to the start of the projection. This reversal happens because of continued improvements in educational attainment. As the improvement in educational attainment slows down around 2035, per capita GDP growth declines again and transitions monotonically towards its steady-state value of 1.5\%.

The continued aging of the population well into the future generates a continued rise in the capital-to-labor ratio. Consequently, real interest rates continue to decline an additional percentage point over the 30-year projection horizon. In the long run, interest rates turn negative, eventually
reaching a steady-state value of -0.43%.

5 Evaluating the Mechanism

5.1 Decomposing Growth

Over the entire transition path, the only values that are varying in a non-trivial way are age demographics (driven by birth and mortality rates), educational attainment, and immigration flows. Consequently, any variation in trends must result from changes in these factors.

Figure 8(a) decomposes output growth ($g_Y$) into weighted growth rates of capital ($\alpha g_K$) and productive labor ($\alpha (1 - \alpha) g_L$). Both capital and labor growth decline in the first decade, likely
understating the real-world values. Labor growth then increases until the mid-1980’s as the baby boom generation enters the workforce. The labor growth rate then remains elevated as college attainment continues to improve, but the growth rate declines persistently through the remainder of the time period. Capital growth increases steadily from 1970 until around 2000 as the baby boom generation accumulates wealth. The increase in capital growth over that time period offsets some of the contemporaneous decline in labor, leaving output growth relatively stable at an elevated rate. Then, the decline in labor and capital growth after 2000 accelerated the decline in output growth over that period.

Figure 8(b) decomposes output per capita growth \((g_y)\) into total output growth \((g_Y)\) and population growth \((g_p)\). The population growth rate declines sharply over the first two decades, while total
GDP growth rises. These two effects combine to elevate per capita GDP in the 1970’s. Between the early 1970’s and the early 1990’s, the population growth rate actually rose slightly, while the total GDP growth rate declined. These trends resulted in a decline in per capita GDP throughout those decades. Then, the population growth rate declined again, as total GDP growth continued to decline. Since total GDP declined at a faster rate than the population growth rate, per capita GDP continued to decline through the end of the series.

Figure 8(a) shows that the capital growth rate declines in the last two decades, showing a similar pattern as investment growth in Section 4.3 At the same time, capital as a share of output rises. The relative increase in savings happens for two reasons related to the aging of the population. First, a decline in the fertility rate shifts the age distribution upwards, increasing the share of individuals near the peak of lifetime savings. Second, an increase in life expectancy further shifts up the age distribution. The increase in life expectancy also increases lifetime savings, as more savings is required to smooth consumption later in life.25

A relative decline in labor flows as the capital stock rises causes the capital-to-output ratio to rise throughout most of the series. The abundant capital supply, in turn, causes a persistent decline in the interest rate. This again points to the role of demographics as the cause of capital deepening in discussions of secular stagnation.

5.2 The Role of Education

To measure the impact of education on the economy, the skill premium is set to zero (i.e., $\epsilon^*_t = 1 \forall (e, t) \in \{h, l\} \times N$), and the baseline is solved again. This exercise explicitly assumes that educational attainment generates the improvement in productivity, rather than education being the result of higher productivity, as in Spence (1973). Consequently, these results likely generate an upper bound on the effect of college attainment.

Much of the rise in college attainment started after 1950, when the college attainment rate was only 6%. Figure 9 shows that because of the low college attainment rate, the impact of college

\[25\text{De Nardi, French, and Jones (2009) evaluate this mechanism and find a strong positive relationship between life expectancy and savings.}\]
attainment on the economy was minimal in that year. In particular, without college attainment, the model estimates that GDP 1950 would have been 5% lower. As the college attainment rate improved after 1950, the impact on the economy grew with it. By 2019, the college attainment rate lapsed 30%, increasing annual GDP by 18% in that year. Improvements in college attainment rates also generate an increase in the growth rate of GDP. The results show that, since 1970, educational attainment has contributed at least 0.25 percentage point to the annual growth rate.
of GDP. Given an average GDP growth rate of around 3% since 1970, the results suggest that educational improvements accounted for roughly 8% of economic growth. The model also assumes no relationship between education and age demographics (i.e., fertility and mortality), resulting in no change to the population, and making the effect on GDP per capita the same as total GDP.

Figure 9 also shows how education affects both factors of production, with each declining more than 15%. This suggests the impact of educational attainment on the economy materializes through enhanced capital supply, as well as labor productivity. Without college attainment, the decline in productive labor would be higher than the decline in capital, leading to a heightened capital-to-labor ratio, ultimately causing a 20 basis point decline in interest rates. Finally, the decline in capital and labor would reduce capital and labor income, leading to a decline in personal income tax revenue of 25% in 2019.

5.3 The Role of Immigration

To understand how immigration restrictions would have affected growth over the transition path, the immigration rate is set to zero in every period, i.e., $\psi_t = 0 \ \forall \ t$. The initial steady state corresponds to the year 1850, so the cumulative effect on the population and economic aggregates begins in that year.

Without any immigrants since 1850, Figure 10 shows that the U.S. economy would have been around 80% smaller in 2019. The impact of immigration on the growth rate of the economy would have been small in 1950, when the share of immigrants in the baseline was nearing a trough. Since that time, the impact on the annual GDP growth rate increased from around 0.1 percentage points to around 0.6 percentage points in 2019, accounting for over 20% of economic growth in that year.

For most of the time since 1950, GDP per capita would be slightly lower without immigration. In between 1990 and 2010, the immigration rate rose above its trend. Since the average educational attainment of immigrants in that time period was lower than that of natives, the immigration wage reduced average productivity, reducing GDP per capita. In the final decade, however, immigration rates reverted to normal levels, but the educational attainment of immigrants began rising sharply. Without immigration, average educational attainment of the population would have fallen, as shown
in Figure 10(n) causing GDP per capita to decline more rapidly starting around 2010. A similar argument holds true for the growth rate of GDP per capita.
Since most immigrants arrive well below retirement age, immigration tends to skew the age distribution downwards, leading to a reduced retiree dependency ratio, relative to the baseline. Without immigration, the age distribution skews upwards, increasing the capital-to-output ratio and reducing interest rates by roughly 20 basis points since 2000.

5.4 The Role of Taxation

![Graphs showing the effects of taxation on various economic indicators.](image)

Figure 11: Effects of Taxation on the Economy.
Figure 11 shows how the baseline changes with all taxes set to zero. The results show that, to raise revenue equal to around 6% of GDP, about 3% of output is sacrificed. Further, the results highlight level effects but almost no sustained growth effects resulting from taxation in the model. An interesting finding is that, in the absence of variation in tax rates along the transition path, capital experiences a greater incidence of taxation over time. This makes sense, as the demographic shift leads to a relative increase in capital over time.

In 2019, an economy without taxes would have delivered a capital stock around 8.25% higher and productive labor around 0.34% higher, resulting in 3.1% higher output. These estimates seem reasonable from the perspective of the literature. For example, Conesa, Kitao, and Krueger (2009) use a similar model with idiosyncratic risk and find aggregate changes around this magnitude resulting just from budget-neutral tax reforms. Taxes are shown to have a disproportionate impact on capital, relative to labor. As a result, without taxes, the capital stock would be higher, relative to labor, causing interest rates to be around 50 basis points lower.

6 Policy Reforms

To understand the implications of immigration and tax policy, three reforms are evaluated. Each reform is relatively extreme in magnitude for the purpose of illustration. The first reform is an increase in the immigration rate, while the second reform increases the share of skilled immigration. Finally, the third reform is an aggressive personal income tax cut. Each reform is an unanticipated change implemented in 2019.

6.1 Immigration Policy

6.1.1 Immigration Rate

The first reform is a fourfold increase in the projected immigration rate in every period. This reform preserves both the projected education shares and age distribution of immigrant flows.

\footnote{The volatility in the corresponding graph is just a consequence of plotting a highly magnified series, highlighting numerical error at that level of precision.}
Mathematically, if \( \{ \psi_{2019}, \psi_{2020}, \ldots \} \) is the sequence of immigration rates starting in 2019, then the policy reform changes these values to: \( \{ 4\psi_{2019}, 4\psi_{2020}, \ldots \} \).

Figure 12: Effects of a 4× increase in the immigration rate, 2020-2070.
The results of the simulation are shown in Figure 12. After the first decade, output increases about 15% as it climbs persistently, and the growth rate of output increases by 1.3 percentage points before declining again. This increase is driven by similar increases in both labor and capital. Heightened labor and capital increases output per capita, and the growth rate of output per capita increases before declining towards a new steady state. Since the college attainment rate had been rising faster for immigrants, relative to natives, at the time of the policy change, heightened immigration rates result in a higher share of the population reaching college attainment.

The increase in immigration skews the age distribution downward, reducing both the retiree dependency ratio and average age of the population. This causes a demographics-driven reduction in the capital-to-output ratio, leading to a reduction in the interest rate throughout the projection window.

6.1.2 Skilled Immigration

Figure 13 shows the effects of maximally restrictive skill-based immigration policy. In particular, the policy keeps the immigration rate constant, but sets the share of college-attaining immigrants to 100%. Consequently, the age distribution for all incoming immigrants equals the age distribution for incoming college-attaining immigrants.

Output rises 3% in the first three decades before falling again. This is driven by a steady increase in both labor, as well as capital, again highlighting investment as a channel through which immigration impacts the economy. The growth rate of output rises sharply before trending back towards zero. The increase in the average productivity of workers eventually increases GDP per capita by 3%. The growth rate of GDP per capita rises initially by 0.2 percentage points before falling back towards zero. As labor and capital improve, so too does government revenue, peaking at a 6% increase after three decades before falling back down.

Because most college attainment happens in a person’s early 20’s, college-attaining immigrants tend to be older. Further, strict interpretation of skill-based immigration policy (as it is implemented in the model) restricts any immigrants below the minimum immigrant college attainment age of 18. Without any immigrant children, the age distribution skews upward, and the retiree
dependency ratio increases.\textsuperscript{27}

\textsuperscript{27}Note that birth rates are a function of the total population. As a result, skilled immigration still leads to more
An older population, relative to the baseline, causes a small demographic-driven increase in the capital-to-output ratio, causing a reduction in the interest rate. The maximally restrictive skill-based immigration policy also has an impact on the total college attainment rate, as it rises by 6 percentage points (relative to the baseline) in the long run.

### 6.2 Tax Policy

Figure 14 shows the effects of a 20% tax cut, or, mathematically, $0.8\phi_{e}(\cdot) \forall e \in \{h, l\}$. For the first decade, annual GDP rises by 0.4%, further rising in the long run by 0.8% of baseline output. The growth rate of output rises minimally by 0.06 percentage point before falling back towards zero. Since the population remains constant, GDP per capita and the growth rate of GDP per capita have the same response as total GDP. In the long run, capital rises by 2%, and labor rises by less than 0.1%. The rise in capital, relative to labor, leads to a small long-term reduction in the interest rate by 10 basis points.

Government revenue immediately falls by 20%. As behavior adjusts, capital and labor rise in the long run, but tax revenue rises only slightly. At its peak following the reform, the fall in tax revenue is 19.8%, suggesting that only 1% of the decline in revenue is offset by an improvement in economic activity. In the long run, however, the improvement in GDP is 0.8%, while the decline in tax revenue is about 1.45% of baseline GDP. This suggests that, locally, roughly 0.55% of GDP is sacrificed for every percentage point increase in tax revenue, relative to GDP. This value can be compared to the exercise in Section 5.4 where the elimination of the income tax reduced revenue by 6.3% of GDP in 2019, but improved output by 3.1%—a total GDP cost equal to 49% of total revenue.

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28 For robustness, a 20% tax increase was also evaluated, leading to similar results in the opposite direction.
29 This observation also confirms the convexity of the marginal GDP cost raising revenue.
Figure 14: Effects of a 20% Tax Cut, 2020-2070.

7 Conclusion

The model presented in this paper contributes to the macroeconomics literature by highlighting the link between demographic changes and the historical paths of several aggregate variables. By including immigration and education in the model, the framework also measured the individual contribution of each to economic outcomes. Finally, the paper contributes to the economic policy literature by evaluating the effects of immigration and fiscal policy reforms.
The model replicated the direction, and in many instances, the magnitudes, of economic trends since around 1970. Of particular interest is the decline in economic growth and the decline in interest rates often attributed to changing demographics. This paper supports those claims and extends the set of observations to include other macroeconomic variables.

Although the model showed a clear link between age demographics and macroeconomic outcomes, the results indicated that education and immigration played significant roles over the last several decades in improving the level of output and the growth rate of output—both on an aggregate and per capita basis. Rising immigration rates and improvements in college attainment since the 1950’s led to sustained growth over the last several decades, highlighting the role that education and immigration could play in the future.

Immigration policy, both skill-based and otherwise, was shown to have an impact on both the level and growth rate of the economy. Fiscal policy, by contrast, was shown to only have a meaningful impact on the long run level of output. Despite having a limited impact in near-term, however, the results suggested roughly one-half of one percentage point in long-run growth is sacrificed for every percentage point increase in tax revenue, as a share of GDP.

While the model generates several observations in the data since around 1970, the model failed to account for high levels of economic growth in the preceding two decades. As mentioned before, this flaw in the model was confirmed by other research. Consequently, it seems likely that other factors account for that period of heightened economic growth. This finding also possibly shows that demographics play an increasingly important role in accounting for macroeconomic trends over time.

Although this paper generated the observed decline in real interest rates within a closed economy, the same outcome may have been produced by a global macroeconomic model. In particular, the demographic changes that led to a declining real interest rate in the model have been a common trend in several OECD countries. Consequently the observed demographic changes in the U.S. may have only contributed to the declining in real interest rates, rather than be the exclusive cause of the trend. In a practical sense, even the most aggressive policies evaluated by the model had a

\[30\text{See, for example, Auerbach et al. (1989) and Carvalho, Ferrero, and Nechio (2016).}\]
limited impact on prices, so the sensitivity of prices to domestic policies may be limited. Future research, however, might address this ambiguity to expand our understanding.

References


