Background Information: Key Elasticities in the Diamond-Zodrow Model

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In this background document, we provide some information on the choices of key parameter values used in the Diamond-Zodrow model. We focus on parameters involving labor supply, saving, international capital flows, and substitution among factors of production.

I. Labor Responses and the Elasticity of Labor Supply

The elasticity of labor supply measures the responsiveness of household labor supply in response to changes in after-tax wages and is an important determinant of the employment effects of any tax policy change in the model. For example, a labor supply elasticity of 0.25 indicates that a 10 percent increase in after-tax wages would result in a 2.5 percent increase in labor hours worked. The observed market or “uncompensated” labor supply elasticity reflects the net effect of two separate effects of a wage increase in a single-period model of consumption and labor supply. The first and the focus of most analyses is the “substitution effect” which reflects the fact that, holding income constant, a wage increase makes working more attractive and results in an increase in labor supply, measured by the “substitution” or “compensated” labor supply elasticity. The second is the “income effect” which reflects the fact that a wage increase implies that for any level of labor supply workers have more income and will spend some of that additional income on more leisure, that is, less labor supply; this response is captured by the “income elasticity,” which is negative under the assumption that leisure is a normal good so that some of an increase in income is spent on more leisure. The sign of the uncompensated labor supply elasticity is thus theoretically ambiguous. However, empirical evidence suggests that the income effect is in fact negative but is outweighed by the positive substitution effect so that the uncompensated labor supply elasticity is positive.
More specifically, in an often-cited review of the empirical literature on labor supply, the Congressional Budget Office (McClelland and Mok (CBO, 2012), which updates a 1996 CBO study) draws the following four main conclusions. First, substitution elasticities for men and single women range from 0.1 to 0.3. Second, substitution elasticities for married women, which earlier studies indicated were considerably larger than those for men and single women have decreased over time as married women have become more important in, and more attached to, the labor force, and now range from 0.2 to 0.4, only slightly higher than the range for men and single women. Third, income elasticities for both groups are small, ranging from -0.1 to zero; thus, the uncompensated elasticities for both groups are positive and there is little difference between uncompensated and compensated labor supply elasticities. Fourth, combining these elasticities implies a total population substitution elasticity ranging from 0.1 to 0.3 and an uncompensated elasticity ranging from zero to 0.3.1 2 In our model parameterization, the substitution elasticity (calculated using the approximate formulas calculated by Gravelle (2007, 2014)) is 0.33, which is just outside the high end of the CBO range. Note, however, that Chetty (2012) argues that many empirical studies understate the labor supply substitution elasticity because they do not take into account labor market frictions – such as adjustment costs or misperceptions of the after-tax wage – that decline in the long run. Correcting for such frictions, Chetty obtains a labor supply substitution elasticity equal to 0.33, identical to the value used in our model. Our income elasticity of -0.24 is, however, somewhat large in absolute value relative to the range noted above, a result

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1 The CBO (2012) study focuses on labor supply estimates obtained from microeconomic studies, typically using tax data or household survey data. Although studies based on macroeconomic data typically generate larger elasticities, CBO argues that micro studies better capture the labor supply effects of permanent tax changes for several reasons, especially since macro studies include cyclical effects and often do not adequately isolate the effects of wage changes.

2 The CBO (2012) study draws numerous other conclusions, including (1) labor supply effects are divided roughly equally between hours worked and labor force participation effects, and (2) the labor supply responsiveness of high-income individuals is roughly similar to that of the general population, but lower-income workers have somewhat higher labor supply elasticities, primarily due to a larger labor force participation response.
that is attributable to the fact that the income elasticity of leisure demand for our household utility function is by assumption equal to one. However, for revenue neutral reforms such as those we analyze, income effects roughly cancel, so the income elasticity is less important than the substitution elasticity in accurately modeling the effects of tax reforms.

Another measure of the responsiveness of labor supply to changes in after-tax wages that is often used in a dynamic intertemporal model such as ours is the Frisch elasticity of labor supply. For our model and most others in this literature in which the intertemporal utility function is time separable, the Frisch elasticity measures the variation in labor supply in a single period in response to a change in the after-tax wage in that same period (only), along an optimal path with the marginal utility of lifetime income or wealth (the multiplier on the lifetime income budget constraint) held constant. It thus captures a pure substitution effect in an intertemporal model that includes the substitution of leisure for consumption over time, analogous to the constant income substitution effect in the standard single-period model.

The relevance of the Frisch elasticity in our analysis is open to question, however, since we are examining a permanent change in the after-tax wage rather than a wage change that occurs only in a single period. In the latter case, individuals are likely to substitute labor across periods; for example, in response to a wage increase in period $t^*$, an individual may work more in period $t^*$ and then work less in period $t^*+1$, when the wage returns to its original level. Because of this intertemporal substitution, CBO (2012a) notes that estimates of Frisch elasticities are typically about 50 percent larger than estimates of the Marshallian substitution elasticity. However, the
incentives for such substitutions would no longer exist for a permanent wage change such as those analyzed in our study.  

In any case, in a survey of estimates of the Frisch elasticity based on micro data, the Congressional Budget Office (CBO (2012a), by Reichling and Whalen) suggests a range of 0.27 to 0.53, with a central estimate of 0.40. The Frisch elasticity in our model of 0.37, slightly below the central estimate.

II. Savings Responses and the Intertemporal Elasticity of Substitution

The response of savings to changes in after-tax rates of return is also a critical determinant of the effects of tax reforms in the model, as changes in saving behavior give rise to changes in investment, the capital stock, labor productivity, wages and living standards. Changes in after-tax returns also give rise to opposing substitution effects (e.g., a lower rate of return increases the price of future consumption and thus makes saving less attractive), and income effects (a lower rate of return makes it more difficult to achieve any given level of future consumption, leaving less money for current consumption, i.e., less saving. In addition, in multi-period models with earnings in more than one period, a third “wealth effect” arises because a lower rate of return implies that a lower discount rate is used to discount future earnings; this increases wealth, which in turn causes individuals to consume more, i.e., save less. The wealth effect thus reinforces the substitution effect. Because the wealth effect typically involves discounting over many periods, it can have a large effect on savings. Indeed, many critics of life-cycle models (and infinite-horizon models to an even greater extent) argue that savings responses are unreasonably large in such models, relative

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3 Thus, Attanasio et al. (2018) note that the Frisch elasticity is best used to analyze the effects of business cycle fluctuations in wages rather than permanent wage changes, and Gravelle (2014) argues that the Frisch elasticity “is not likely to be of importance in an analysis of a permanent tax changes.”
to empirical estimates of savings elasticities with respect to after-tax returns, which typically range from zero to 0.4 (Penn-Wharton Budget Model (PWBM), 2016).

The key parameters in determining the responsiveness of saving to changes in the after-tax rate of return are (1) the intertemporal elasticity of substitution (IES), which reflects the change in the ratio of consumption in two periods in response to a change in the relative prices of consumption in those periods, which is determined by the after-tax rates of returns in those periods, and (2) the personal rate of time preference, which reflects the rate at which households discount future utility.

PWBM (2016) note that estimates of the IES center around 0.33 and that Elmendorf (1996) shows that such a value yields savings elasticities that range from 0.03 to 0.4, consistent with the empirical data noted above. There is little empirical evidence about the personal rate of time preference, although most studies assume rates that range from zero to 0.02 (although negative rates of time preference are also possible and sometimes used). Elmendorf (1996) shows that the use of combinations of intertemporal elasticities of substitution and rates of time preference in these ranges in a life-cycle model typically yield savings elasticities in the neighborhood of 0.65, somewhat higher than the empirical range noted above.

We use a somewhat higher IES of 0.5, which tends to give us higher savings responses, as does our choice of a rate of time preference of 0.15, toward the high end of the range for that parameter. However, this tendency is offset by the fact that our model has a target bequest motive, under which households save to finance a fixed bequest. Such bequest behavior reduces the response of savings to after-tax returns in the model, since an increase in the after-tax rate of return reduces the amount of saving required to finance the fixed bequest. Our approach follows Fullerton
and Rogers (1993) who also use an IES of 0.5 in the context of a model with target bequest behavior.

III. The Elasticities of International Capital Flows

Another key parameter in the model is the elasticity of supply of foreign capital to the United States, as international capital flows tend to accentuate the effects of changes in after-tax returns. For example, if a tax reform reduces the after-tax return to capital in the United States, not only will domestic saving and investment decline but international capital will flow out of the country as well. The magnitude of the latter effect is determined by the elasticity of supply of foreign capital. Indeed, if capital is perfectly mobile, capital will flow out of the country until returns to capital go back to their original level. More generally, a larger elasticity of supply of foreign capital implies a smaller reduction in the after-tax return to capital, a larger reduction in the domestic capital stock, and thus lower labor productivity and lower wages in the United States.

Gravelle and Smetters (2006) review the literature estimating the capital supply elasticity and argue that it suggests an aggregate value between 1.0 and 3.0. However, recent work by Mutti and Ohrn (2019) and Steinmuller, Thunecke, and Wamser (2018) suggests somewhat less mobility of international capital investment in response to tax differentials than previous research. For example, Mutti and Ohrn estimate an elasticity of international capital investment (expressed as a positive number) with respect to average effective tax rates of 0.7 to 1.8. We have two types of capital in our model – relatively immobile ordinary capital and more highly mobile firm-specific

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capital, so we must choose two capital supply elasticities. Drawing on the Mutti-Ohn analysis, we assume that the supply elasticity of ordinary capital is 0.5 and the supply elasticity of firm-specific capital is 3.0.

IV. The Elasticity of Substitution in Production

Finally, the elasticity of substitution between capital and labor in production measures the extent to which capital and labor can be substituted in production, specifically the percentage change in the capital-labor ratio in response to a percentage change in the factor price ratio. For example, if a tax reform increases the cost of capital, the elasticity of substitution between capital and labor in production measures the ease with which labor can be substituted for capital; in this case, a large value implies the employment effects of the tax change will be moderated as labor will be substituted for capital to a relatively great extent.

The debate in the literature has focused on whether the elasticity of substitution between capital and labor in production is one – which is the case of the Cobb-Douglas production function – or a value less than one. In a recent review of the empirical literature on this issue, Knoblach, Roessler and Zworschke (2019)\(^6\) examine 77 studies of this issue published between 1961 and 2017. They estimate that the long run elasticity of substitution between capital and labor in aggregate production varies from 0.45 to 0.87. Primarily in the interest of simplicity, especially since our model includes relatively complex representations of investment in the presence of adjustment costs, we assume Cobb-Douglas production functions in all of our production sectors and thus assume an elasticity of substitution in production of one, somewhat above the top of the

range noted above. Although this assumption implies that the elasticity of substitution in production is overstated, it is an assumption that is often made in the literature.
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