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Automation Does Not Kill Jobs; It Increases Inequality

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Abstract

Many economists have been concerned that automation will result in loss of jobs. This work shows that is not the issue. There are two main effects of automation: increased inequality and economic growth. As the cost of automation drops, capital's share of the economy grows as capital substitutes for tasks of the bottom ninety percent of workers. These workers are competing with automation, which links their wages to the marginal cost of automation. The wages of highly skilled workers grow because they are a complementary factor to automation. Creating high paying jobs for the rest of workers requires that the tasks necessary for these new jobs cannot be performed by automation. Furthermore, the abilities involved in these tasks must be scarce. If this is not the case the supply of workers capable of doing these jobs will increase until the quasi-rents are eliminated.

This work describes a macroeconomic model that describes how the conclusions above were reached. Data from the studies by Frey and Osborne, were used for CES and Cobb-Douglas production functions. Data from the study by Brandes and Wattenhofer were used for the CES production function. The results from these differing inputs were compared and are qualitatively similar, describing progressing economic growth and growing inequality as the cost of automation decreases.

1. Introduction

Since the 1980's, it is an empirical fact that inequality has increased. This is best documented using Federal Reserve data[1] as shown in Fig. 1 below.

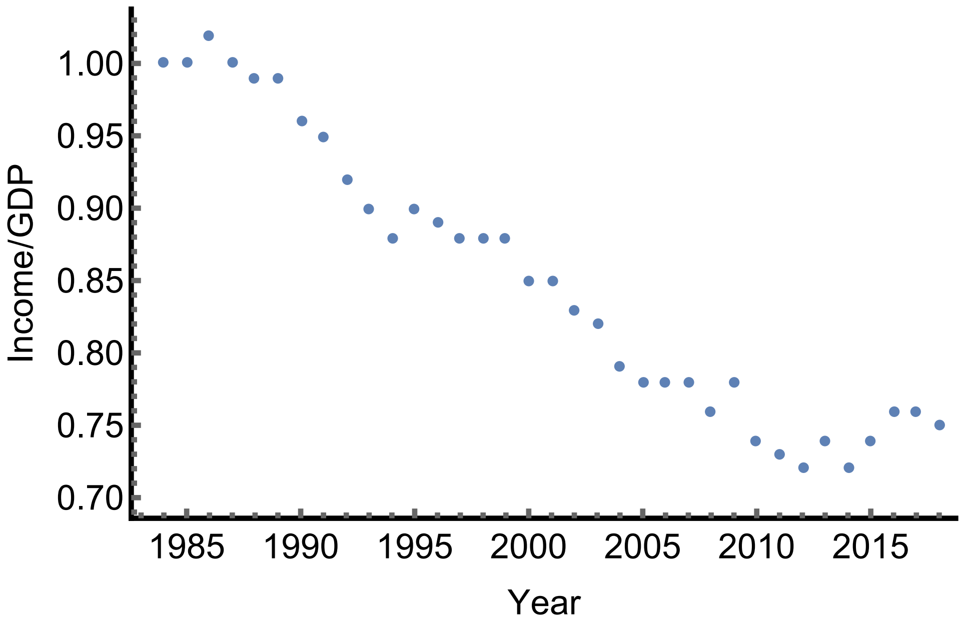

Fig. 1. U.S. median incomes per household divided by per capita GDP.[2]

The same Federal Reserve data showed that GDP per household adjusted for inflation grew by 65 percent from 1984 to 2016. Our model explains this behavior.

Automation in a Historical Context

Until the late eighteenth century, all goods were created and transported by the brains of humans and the muscles of humans and animals. Then machines began replacing the muscles of humans and animals. The division of labor in production resulted in skilled workers being replaced by unskilled workers. Over the next one hundred fifty years the Industrial Revolution transformed the economies and social structures of the Western World. For most of that period, labor struggled to overcome the displacements caused by technological change until finally the enormous gains in labor productivity and the resulting growth of wealth came to be shared with labor. *Now automation is replacing the brains of humans in the creation and transport of all goods.*

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[1] FRED blog (2016)

[2] This is the data of the third graph of the FRED blog (2016) replotted as the lower curve divided by the upper curve. That the initial value of the ratio equals 1 has no significance.

In the 1950's in an exchange between Walter Reuther, President of the United Autoworkers Union, and a Ford official during at plant tour, the official pointed to some automatically controlled machines: [3]

> Official: "How are you going to collect union dues from these guys?"
> Reuther: "How are you going to get them to buy Fords?"

After this conversation, it took fifty years for automation to become a major occupation of the minds of economists. In roughly the last decade, this subject has attracted the attention of a number of workers and papers expressing a variety of opinions ranging from widespread unemployment caused by automation to a new age of economic growth with full employment.

There is general agreement that there are three possible ways in which automation affects the economy and thus society:

1. Increasing the productivity of workers by *augmentation*
2. Creation of new and presumably better jobs by *creation of new industries*
3. Displacement of workers by *substitution*

Augmentation should not lead to the displacement of workers. The other two effects are relatively new. In considering them, the optimist focuses upon the creation of new industries; the pessimist upon substitution. The view of the optimist is that technology has a long track record of creating industries that provide new and often large employment opportunities such as the creation of web site developers or computer support specialists. In this context, the following points should be remembered.

1. These new jobs have to employ lower ability workers in tasks not susceptible to automation.
2. Technology developments can destroy goods and jobs as well as creating them. We point out that experience indicates that while new industries and jobs constantly appear, many goods and services disappear. Appliance repair is no longer

[3] Reuther 1954

economically viable; most consumers no longer purchase a separate camera; floppy disks have become a quaint artifact.

Economists have reacted to the need to study the effects of automation in at least three different ways. Autor and Resterepo have developed[4] a task-based model in which high- and low-skill workers compete against machines in the production of tasks. Automation displaces the type of labor it directly affects, depressing its wage. They assume as we do that capital adjusts to keep the interest rate constant.

A number of economists, probably most significantly David Autor,[5] have looked at the impact of automation by examining what is actually happening, while others specifically Frey and Osborne and McKinsey & Company, have broken down the tasks comprising occupations in order to see which jobs are going to be most impacted. There is considerable disagreement about what will be the skill levels of workers who will be most affected.

Of particular interest to our work are the efforts focused on analyzing the importance of the various tasks needed to carry out particular occupations. This approach was pioneered by Frey and Osborne[6] , McKinsey & Company[7], and Brandes and Wattenhofer.[8] Frey and Osborne devoted their efforts to the prediction of which jobs are susceptible to being automated concluding that a minimum of 47 percent of jobs can be automated. Brandes and Wattenhofer reviewed this work arguing that Frey and Osborne overestimated the susceptibility of certain tasks to automation. While primarily using the results of Frey and Osborne, we have also applied our model to the results of Brandes and Wattenhofer analysis. McKinsey&Co[5] take a different view arguing that for most jobs only part of the tasks required by a job will be automated displacing only some of the workers holding that job. In our work, we assume that Frey and Osborne in their probabilities of automation capture the fraction of the tasks performed by workers in a particular job that will be automated. But in agreement with McKinsey&Co., in the macroeconomic model described

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[4] Acemoglu and Restrepo 2016. 2018.
[5] Autor and Dorn, 2013, Autor, Dorn, and Hanson, 2015, Autor, Dorn, Katz, *et al,* 2017, Autor, 2019
[6] Frey and Osborne, 2012, 2017.
[7] McKinsey & Company, 2017.
[8] Brandes and Wattenhofer, 2016

here the job continues to exist, but employs fewer workers and the nature of the job may be change.

We use a market-based macroeconomic model to explore the effects of automation upon a modern economy. This model differs from most standard approaches in two ways. First, we address technical change at the level of the firms making a particular good. Second, we are assuming that labor is heterogeneous: the labor force is divided into skill groups, highly skilled workers, medium skilled workers and low skilled workers. We designate these as Type A workers, Type B workers and Type C workers respectively.

We must emphasize that we have no way of predicting the rate[9] of technical change so the dynamics of our model is based on the cost of substitutional automation rather than upon time. In his recent book *The Technology Trap*,[10] Carl Benedikt Frey considers what we might learn from the history of past disruptive technologies that could help in creating a timeline for the application of automating technology.

In a competitive economy, we find that capital's share and the share of the high-skilled workers increase at the expense of less-skilled workers. In 1980, the ratio of the income of the top ten percent divided by the income of the bottom ninety percent was slightly greater than one. In order to have a possible observable, we measure the extent of automation by the fraction of Type A workers who are in a sector of the economy in which automation is complete. When this measure reaches about 0.3, our model predicts that the ratio of the income of the top ten percent divided by the income of the bottom ninety percent roughly doubles whether measured in wages or total income (i.e. wages plus returns on capital). This market allocation of income may not be politically viable and it may be necessary to intervene with policies that redistribute income.

2. The Model

In order to generate our test cases, we utilize the 1980 United States economy, which we assumed to be without automation, to calculate basic parameters for our model. We assume that the labor force is fixed at the 1980 level. We do not model the capital market and

[9] Although there is no way to predict the speed of technical change it should be noted that the quality adjusted price of robots in six countries dropped by a factor of five between 1990 and 2005. See Graetz and Michaels 2018.

[10] Frey, 2019

assume that capital is can be rented at a fixed rate, q. This assumption implies that we are modeling only the consumption sector of the economy.[11] We assume that the firms have C.E.S. production functions with an elasticity of 0.5. We calculate factor shares that result from a competitive market ignoring governmental actions such as taxes and transfer payments because to do so would complicate an already complicated paper. We postpone that problem to possible later work.

We develop a model where labor is heterogeneous and technical change occurs at the firm level. We use a model similar to Mirrlees[12] and assume the technology is defined by firms distributed uniformly on the interval. Firms are identical except for their cost of using automation, which increases as n increases. We define a "*robot*" (which might be just a computer program) as the amount of automation sufficient to displace a person. We assume that the n-firm has C.E.S. production function with an elasticity of 0.5 [13]

$$(2.1)\quad y(n)=\frac{A}{\frac{a}{k(n)}+\frac{b}{l_A(n)}+\frac{c_1}{l_{B1}(n)+r_B(n)}+\frac{c_2}{l_{B2}(n)}+\frac{d_1}{l_{C1}(n)+r_C(n)}+\frac{d_2}{l_{C2}(n)}}$$

where $k(n)$ is the amount of regular capital, and $l_A(n)$ is the number of Type A labor worker-years. Type B labor can be employed in role $l_{B1}(n)$ in which role it can be replaced by automation, $r_B(n)$, or in role $l_{B2}(n)$ in which it cannot be replaced by automation. $r_B(n)$ is the number of B robots used by the nth firm (reiterating: a "*robot*" is the amount of automation that replaces one *"B"* worker-year). We treat Type C labor in the same way. The distinction between B_1 and B_2 or between C_1 and C_2 workers is only that of current job assignment. The firms are ordered by increasing cost of automation as n increases. We introduce a parameter β, initially set to unity, that we decrease to allow automation to progress. The cost of a B robot to firm n is equal to $q(\alpha_{B0}+n)\beta$ and the cost of a C robot

[11] Because we are ignoring the investment sector, our model has little to do with the actual U. S. economy in 1980, but U.S. 1980 does provide a cost of capital, initial GDP, and number of workers.
[12] Mirrlees 1971
[13] This form is similar to that described by Acemoglu and Autor 2011, page 1104 except that they regard technology as augmentation of the worker rather than replacement of the worker.

to firm n is $q\left(\alpha_{C0}+n\right)\beta$ where $\alpha_{B0}=\frac{w_B}{q}$ and w_B is the preautomation wage of a B worker. Similarly, $\alpha_{C0}=\frac{w_C}{q}$ where w_C is the preautomation wage of a C worker. The distinction between B_1 and B_2 or between C_1 and C_2 workers is determined only by *current* job assignments; B_1 and B_2 are in the same market. Likewise, C_1 and C_2 are in the same market.

The economy is divided into three sectors. The division point between the first two sectors is n_C and the division point between the second and third sectors n_B.

Summarizing the nature of the three sectors that develop:

1) If $n<n_C$ Type A, Type B_2, Type C_2 workers and both B and C robots are employed, but no Type B_1 or Type C_1 workers are employed;[14]

2) If $n_C<n<n_B$ Type A, Type B_2, both type C_1 and C_2 workers and B robots are employed, but no Type B_1 workers are employed;

3) If $n_B<n<\overline{n}$ Type A, Type B_1, Type B_2, Type C_1 and Type C_2 workers are employed, and no robots are employed; $\overline{n}$ is the total number of firms.

The wage of C workers is determined by cost of a C robot, $q\left(\alpha_{C0}+n_C\right)\beta$, at the transition point n_C between Sectors 1 and 2 and the wage of B workers by the cost of B robot, $q\left(\alpha_{B0}+n_B\right)\beta$, at the transition point n_B between Sectors 2 and 3. As β decreases as the result of technical advances, more firms will be able to substitute robots for B and C workers and both n_B and n_C will increase while wages of type B and C workers decrease. As automation progresses, the value of n_B can reach $\overline{n}$, while $n<n_C$ is still the case. We call the situation which $n_B<\overline{n}$,Case 1, and Case 2, the case where n_B is equal to $\overline{n}$ and $n_C<\overline{n}$.

For simplicity we assume that each firm is producing a different good, but the firms differ only in the identity of the good and the cost of robots. Since the production function is linearly homogeneous, it is harmless to assume that only one firm, the nth firm, produces

[14] Please keep in mind that the subscripts refer to the current assignment of workers. B_1 and B_2 workers are still in the same market, and C_1 and C_2 workers are together in their own market.

the *n*th good. We assume that the capital market is always in equilibrium and that the cost of capital is constant justifying this by the assumption that the capital market is global.

We do not model the demand side of the economy and instead assume that there is a representative consumer with a utility function[15]

(2.2) $$W = \int_0^{\bar{n}} \alpha \, ln\left[y(n)\right] dn$$

This formulation of the demand side causes the *value*, α, of every good produced to be the same. We define α by choosing a good made only by humans as the numeraire good.

The labor resource constraints are:

(2.3) $$\bar{L}_A = \int_0^{\bar{n}} l_A(n) dn$$

(2.4) $$\bar{L}_B = \int_0^{\bar{n}} \left[l_{B1}(n) + l_{B2}(n)\right] dn$$

(2.5) $$\bar{L}_C = \int_0^{\bar{n}} \left[l_{C1}(n) + l_{C2}(n)\right] dn$$

$\bar{L}_A$, $\bar{L}_B$ and $\bar{L}_C$ are the amounts of Type A, Type B and Type C workers available. The subscripts represent current job assignments. In the optimization of production, Eqs. (2.3), (2.4) and (2.5) must be satisfied.

It proved possible to eliminate a combination of Eqs. (2.3) and (2.4) reducing the problem to the determination values of n_B and n_C satisfying Eqs. (2.3 and 2.5), The numerical solution for n_B and n_C is challenging because, n_B and n_C, interact in nonlinear ways. We were able to solve this problem by starting at no automation (β=1) and slowly decreasing

[15] This is a common assumption in the literature. See for example Acemoglu and Restrepo 2017, 2018, or Gregory, Salomons, and Zierahn 2016.

β in small steps of $\Delta\beta$ = -0.0005. For these small steps in β, dependences of the search parameters upon β were sufficiently close to linearity that quadratic and higher order terms were small enough to make repeated iteration of the two linear equations obtained by Taylor series expansion of the equations (2.3) and (2.5) converge. Further discussion of the solution method can be found in Appendix B. The Mathematica Notebooks together with example results are available in the Supplemental Material.

The CES model has a production function with elasticity ½. In order to gain some insight into the effects of elasticity, we also explored the behavior of a Cobb-Douglas production function with elasticity equal to one.

$$(2.6)\ y(n) = A'\left\{k(n)^a\, l_A(n)^b \left[l_{B1}(n)+r_B(n)\right]^{c_1} l_{B2}(n)^{c_2} \left[l_{C1}(n)+r_C(n)\right]^{d_1} l_{C2}(n)^{d_2}\right\}$$

Unlike the C.E.S. case, the optimization problem for Cobb-Douglas can be solved analytically as described in Appendix A.

3. Computational choices

3.1 Basic parameters of the model

As just indicated, we primarily assume a C.E.S. production function as in EQ. (2.1) with ρ = -1 (an elasticity[16] of σ = 0.5) based upon Frey and Osborne, but also explore this C.E.S choice using the results of Brandes and Wattenhofer and a Cobb-Douglas production function based upon Frey and Osborne.

In order to obtain plausible values for the five constants in the production function, we choose to be guided by the US economy of 1980.[17] In that year, there were approximately one hundred million workers. Prompted by Piketty's data,[18] we set capital at four times GDP. If we assume that this economy is equilibrium, this sets capital's rate of return, q,[19]

[16] See Chirinko, 2008 for a discussion of the estimates of elasticities in the literature.
[17] 1980 was the final year of a long period of constant inequality for the U.S. economy. After 1980, inequality grew.
[17] See Piketty Figure 8.5 p.291
[18] See Piketty Figure 4.6 p.151.
[19] From Piketty's estimate that capital is four times GDP. We have found that the results described here are not sensitive to the assumed rate of return on capital or to the choice of the number of firms, $\overline{n}$.

at 0.0833, assuming no inflation (our model is not concerned with inflation). We set capital's share of production to 0.33333, Type A workers' share is 0.22222 of production, Type B receiving 0.31111, and Type C receiving the remaining 0.13333 share.[20] We are assuming that the firms are leasing capital at a fixed rate.

This model differs in an important way from the actual 1980 economy because we assume that the firms in the economy are all producing consumer goods omitting the firms producing capital and the investment sector.

These assumptions suffice to determine *a, b, c,* and *d* with c and d defined in the paragraph immediately below. The parameter, A, is determined by the reported GDP for 1980.

3.2 Incorporating the results of Frey and Osborne

We assume that there is initially no automation. In order to use the results of Frey and Osborne, we need introduce four new parameters to distinguish tasks from jobs. Thus, c splits into c_1 and c_2 and d splits into d_1 and d_2. In the C.E.S. case $c=\left(\sqrt{c_1}+\sqrt{c_2}\right)^2$ and $d=\left(\sqrt{d_1}+\sqrt{d_2}\right)^2$. The 1980 data determine only c and d, providing no guidance concerning the individual quantities c_1, c_2, d_1 and d_2. In order to determine the individual parameters, we turn to the work of Frey and Osborne (2017) to obtain estimates of c_1 and d_1. In their work Frey and Osborne have analyzed 702 jobs aided by the 2010 O*NET tables. The O*NET table describes and rates the qualities needed to execute the tasks involved in job. The Appendix of Frey and Osborne's work is a table listing the 702 jobs that they analyzed. The second column of that table provides a *probability* that the job will be automated. Since there is a one-to-one correspondence between *fractions* and probabilities, we choose to interpret Frey and Osborne's probability to the fraction of tasks weighted by the amount of human labor involved in the task. Each job is also listed by the code assigned to it in the "*2010 National Employment Matrix title and code*"[21] of the U.S.

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[20] This choice of shares is admittedly arbitrary except for capital's share. Other choices have been explored (see Supplemental Material) giving different results, but all give stagnant wages with GDP growth.
[21] Bureau of Labor Statistics (2011).

Bureau of labor statistics. This BLS table lists the number of persons employed in each particular job and the median wage of these workers.

By combining the BLS data with Frey and Osborne's Appendix table, which also lists the BLS code for each job, we have the means to aggregate the properties of each of our three kinds of labor. Consider for example, the parameter d_l. The quantity, d_l, can be aggregated by determining the 50 per cent of jobs with lowest wages and then calculating the fraction of low wage workers susceptible by averaging the fraction of tasks for a job, f_i, from Frey and Osborne weighted by the number of workers in the job, n_i, from the BLS.

$$\langle f_C \rangle = \frac{\sum_{1}^{nw/2} f_i n_i}{nw/2} \tag{3.1}$$

Where *nw* is the total number of all workers. The same can next be done for the middle 40 per cent of workers by wage $\langle f_B \rangle$ and finally the top 10 per cent by wage to obtain $\langle f_A \rangle$. The results are[22] $\langle f_C \rangle \approx 0.8$, $\langle f_B \rangle \approx 0.45$, and $\langle f_A \rangle \approx 0.1$ implying that about 80 per cent of C workers would be potentially[23] directly impacted by automation loss of job, about 45 per cent of B workers could be similarly impacted, and 10 per cent of A workers. The B and C workers who are not directly displaced by automation are equally impacted because all B and C wages are set by the cost of automation.

In order to earn the wage of an A worker, a person must have *scarce* skills that cannot be automated. The fraction of the tasks of an A level job that can be automated is small with an average value of 0.1. However, upon examining specific A jobs, there are exceptions. Workers such as airline pilots and nuclear power plant operators are primarily employed in case of an emergency and are likely to be safe from displacement as are judges, and other jobs that require the exercise of authority over others. Overall, the indirect effects of automation are expected to be different for high level skills. In general, A level workers will not be greatly adversely affected by automation and we ignore task shrinkage for A workers in our treatment.

[22] Because the jobs listed are discrete, there is never a job exactly at the borders of the wage continuum.
[23] In fact, in our analysis percentage wise B workers are more affected than C workers.

4. Results

In the series of graphs that we show below graphs over the region in which $n_B < \overline{n}$ (Case 1) and the region $n_B = \overline{n}$ and $n_C < \overline{n}$ (Case 2). For Frey and Osborne we found that $c_I = 0.45$ and $d_I = 0.80$ while for Brandes and Wattenhofer we find $c_I = 0.36$ and d_I=0.55. For three of the figures below most concerned with our primary conclusions, we will show results for both of these two sets of parameters. We label Frey and Osborne plots with FO; Brandes and Wattenhofer with BW.

4.1 Factor Shares

Our model predicts that as capital substitutes for medium- and low-skilled workers, capital's share in production will grow and the shares these workers decline. This is shown for our C.E.S. function in Figure 2 below for Frey and Osborne's calculations, in Figure 3 for Brandes and Wattenhofer's calculations and for Cobb-Douglas using Frey and Osborne in Figure 4. For C.E.S., Type A's share grows with growth of the economy caused by capital investment. In all the figures below, the abscissa is the percentage of all A workers employed in Sector 1, which contains both B and C robots. We regard the way A is deployed by the market as real; we regard β as a merely a device to drive automation.

We begin by plotting factor shares over Case 1 (recall that Case 1 is the regime over which B1 tasks become completely automated) beginning with comparing our results using the Frey and Osborne (FO) calculations with our results using the Brandes and Wattenhofer calculations and lastly the FO based Cobb-Douglas production function.

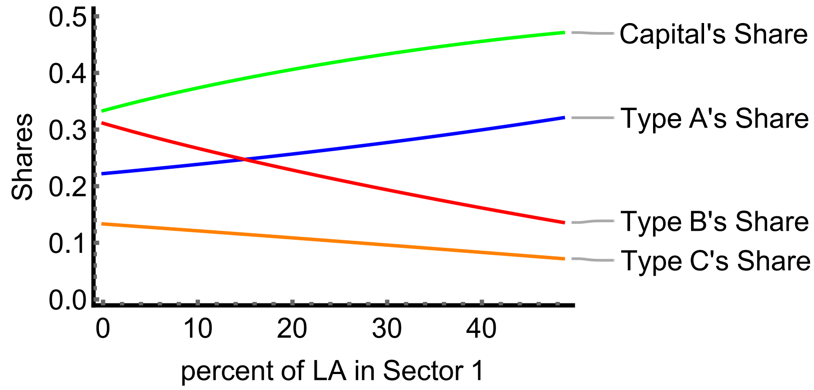

Fig. 2. Shares of the four factors change as automation progresses (C.E.S. FO).[24]

From our calculations based upon Brandes and Wattenhofer with the same scale.

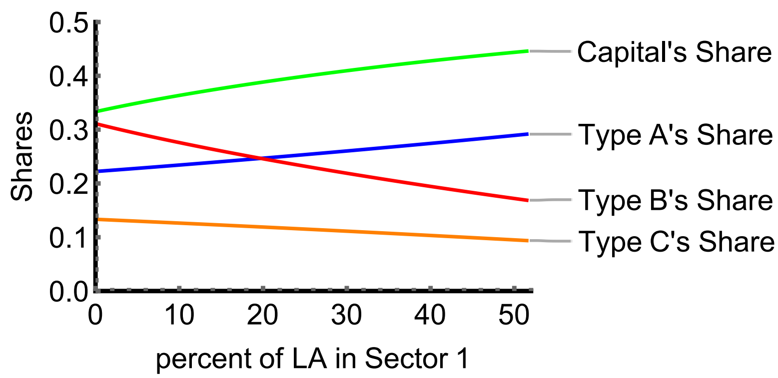

Fig. 3. Shares of the four factors change as automation progresses (C.E.S. BW).

The substitution process is more intuitively obvious in the Cobb-Douglas case

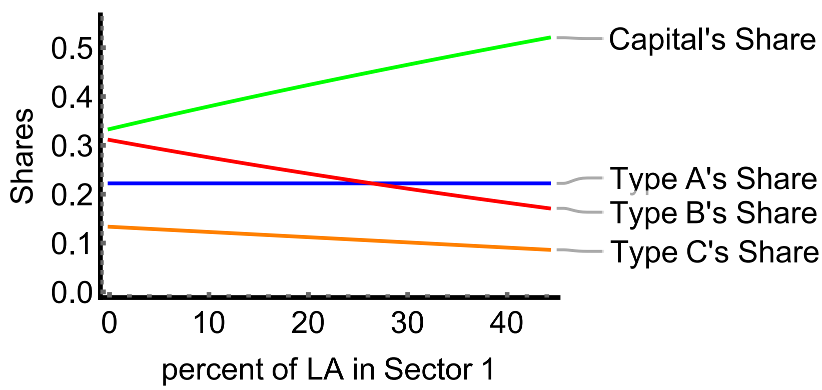

Fig. 4. Shares of the four factors change as automation progresses (Cobb-Douglas model, FO).[25]

The reader will note that L_A's share is constant for Cobb-Douglas as is to be expected. This makes clear that the declines in B's share and C's share in the first two graphs are the result

[24] This is a plot over Case 1 only. Note that the Case 1 LA limits differ among the three figures.

[25] Only Case 1 is included in this graph for comparison with CES

of the substitution of capital for labor. Recall that the wages of B and C labor are determined by the cost of automation, which is decreasing.

At the right extremes of Figures 2, 3, and 4, automation has progressed to the point at which displacement of B workers is complete, i.e. all B workers are employed in jobs where they no longer are in competition with B robots. However, automation of jobs held by C workers is far from complete. Referring back to Section 2, we call the automation region in the previous figures Case 1 and the extended region Case 2. This further extension of automation of C workers changed the nature of the mathematical problem, which was solved by a variation of the approach already used. In these calculations, the numeraire is still the good made the firm at $\bar{n}$ although that firm now uses Type B robots. Figure 5 depicts the C.E.S. factor shares over the entire regime for Frey and Osborne. Figure 6 depicts the C.E.S. factor shares over the entire regime for Brandes and Wattenhofer and Figure 7 depicts the Cobb-Douglas factor shares over the entire regime for Frey and Osborne.

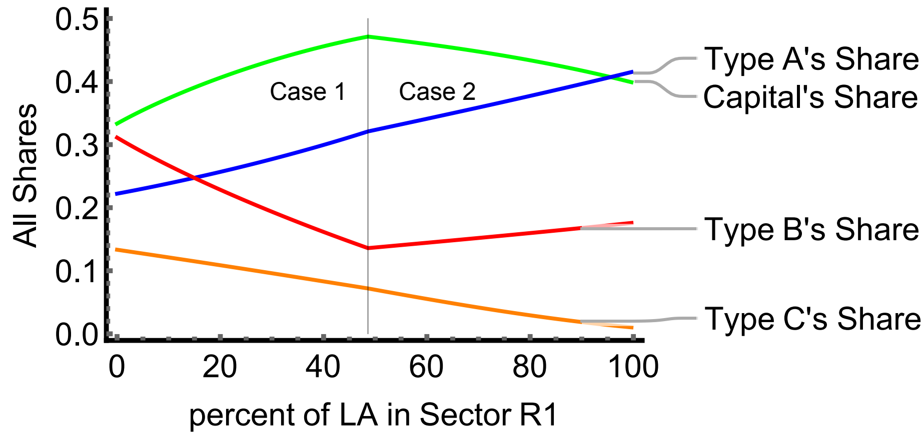

Fig. 5. The effect of completing automation of C workers for C.E.S FO. The Case 1 region to the left of the divider near 50% is the case already shown in Fig. 2.

Note that capital's share is decreasing while A and B's share increases.

The above is compared with Brandes and Wattenhofer in the figure below.

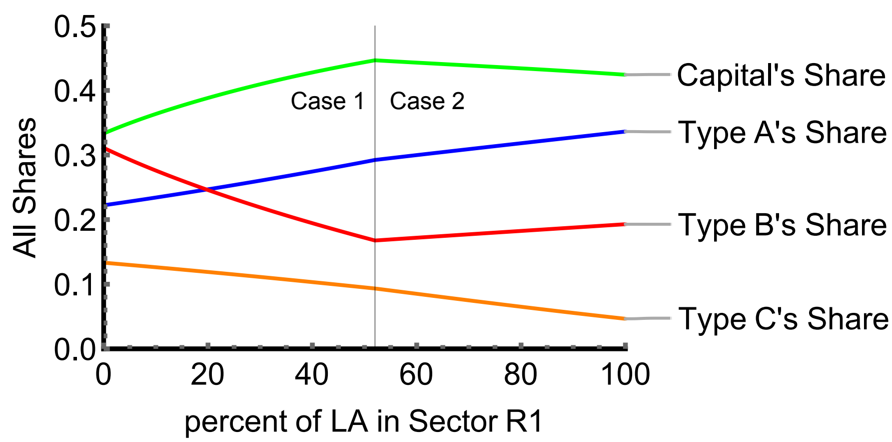

Fig. 6. The effect of completing automation of C workers for C.E.S BW.

Next Cobb-Douglas using Frey and Osborne.

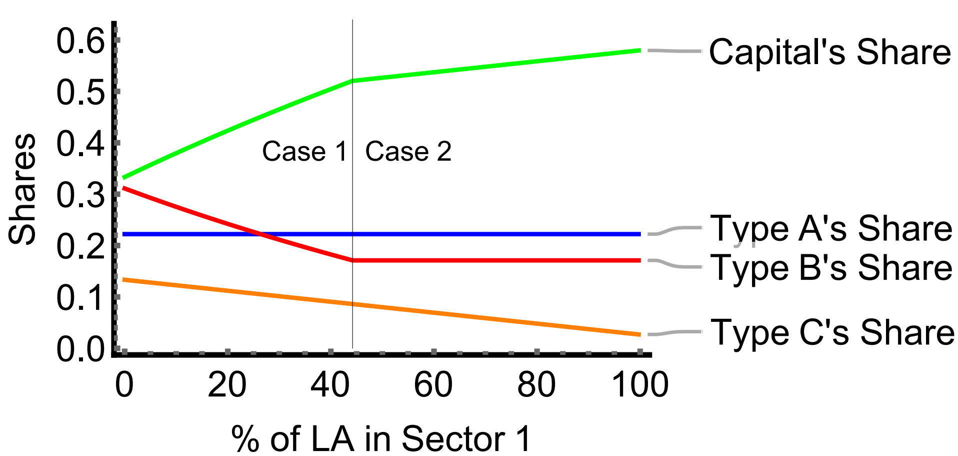

Fig. 7. Completing automation for Cobb-Douglas FO.

For Cobb-Douglas where the elasticity is 1, the share of any factor that is constant is fixed. Thus, A's share is the constant 2/9 throughout. In Case 1, both B and C workers in role #1 are being replaced by automation and their share is decreasing. At the division point between Case 1 and Case 2, replacement of B workers by robots is complete and B workers' share becomes fixed at the share of workers in B_2 roles (0.1711), but C workers continue to be replaced by capital so that C workers' share continues to decrease while capital's share increases.

In contrast, the C.E.S. function chosen has a lower elasticity (0.5), which causes the shares of scarce factors fixed in amount to increase as capital investment increases. The number of A workers is fixed throughout so that an A workers' share increases because robots and A workers are complements in production. Consequently, the marginal product of A workers increases. In Case 1, both B and C workers are substitutes for robots and their marginal product decreases because their wages are tied to robot competition. Thus, the shares of B and C decrease in Case 1. However, in Case 2 with the replacement of B workers complete, B workers are no longer competing with robots, their marginal product is no longer tied to B robots, and they become complements to other factors with the result that the marginal products of B workers grow.

In Case 2 for C.E.S., the market for capital used in B worker automation is saturated. We find that capital is investing in C robots. However, the C workers being displaced get a lower wage making investment in these robots less attractive. For C.E.S., this results in a decrease in capital's share in Case 2 although capital's income continues to increase as output continues to grow.

4.2 Inequality

According to Piketty, the top 10 percent of the population owns 80 percent of the capital in the United States.[26] This means that if we consider income from capital, inequality is much greater than would be suggested by wage income alone. In Figures 8, 9, and 10, both the relation of the shares of A worker's wages to the sum of B and C workers are considered and a similar relation including capital as well.

[26] Piketty p. 348.

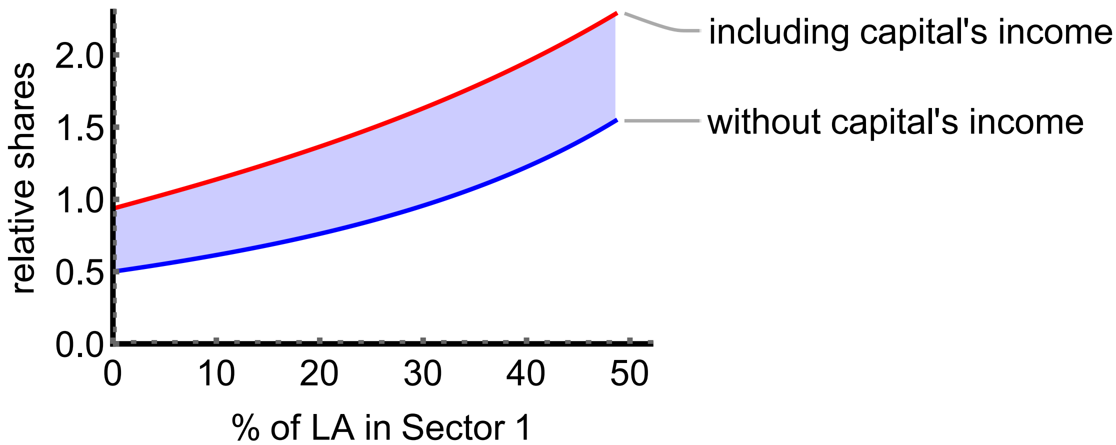

Fig. 8. Ratio of L_A's share to the sum of shares L_B+L_C and the comparison of the sum capital's share and L_A are compared (C.E.S. FO) for Case1.[27]

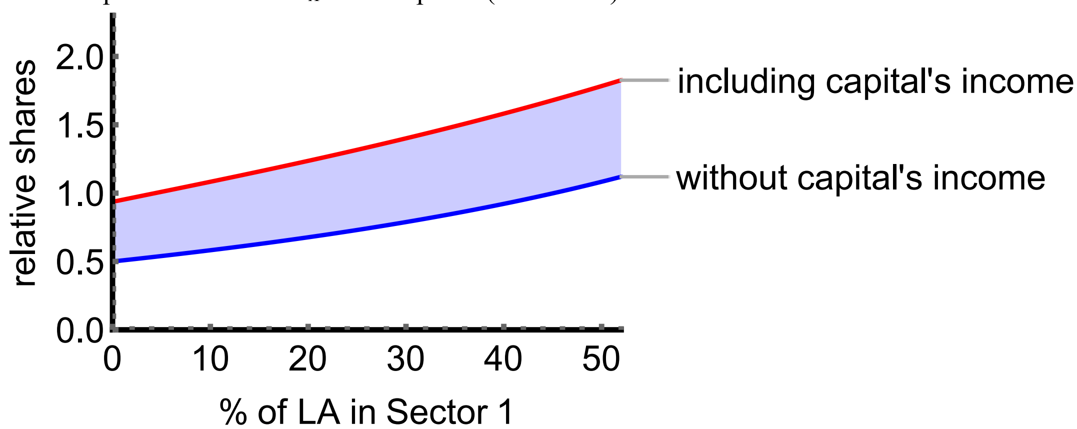

Fig. 9. Like Fig. 8 with the same axes scale using BW parameters.

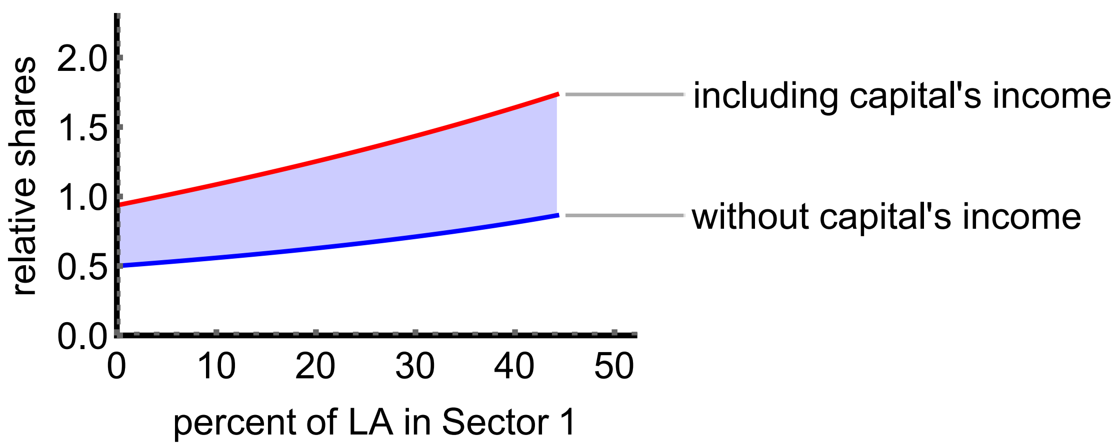

Fig. 10. Ratio of L_A's share to the sum of shares L_B+L_C and the comparison of the sum capital's share and L_A are compared for Case 1 in the Cobb-Douglas model using FO. (The axes scale is the same as Fig. 8)

[27] U.S. citizens own 95 percent of U.S. wealth. The top ten percent own 80 percent of U.S. wealth. See Piketty pp. 156 and pp. 348. The "Including Capital's Income" line may become steeper because as B and C wages decrease these workers will be less able to invest.

For CES Frey and Osborne at full automation of B labor, inequality grows by a factor of more than 2.3 from our hypothetical value of near one for the automation free economy. The inequality is less extreme in CES Brandes and Wattenhofer and Cobb-Douglas Frey and Osborne as shown in Figures 9 and 10 with inequality in Cobb-Douglas FO slightly greater than CES BW.

Another way to examine inequality trends is to plot the labor sector GINI as shown in Figure 11.

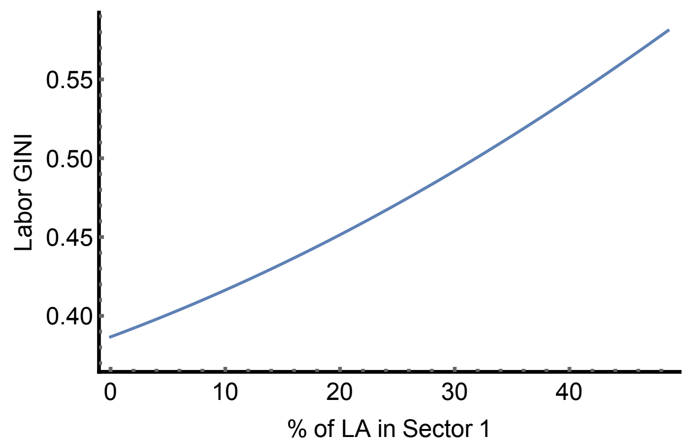

Fig. 11. GINI for the labor wages for Case 1 (CES, FO)

4.3 Welfare

With automation, larger quantities of goods are being produced. However, automation results in changes in factor shares of workers because B and C workers are being displaced by capital. Can the resulting growth of production offset the decreasing wages of B and C workers? To determine this, we used the Pareto test: given two allocations, 1 and 2, a person is better off at allocation 2 if he can buy what he bought at allocation 1 with allocation 2 prices and income. Figure 12 demonstrates that neither B or C workers can buy the same package of goods after automation that they could before automation.

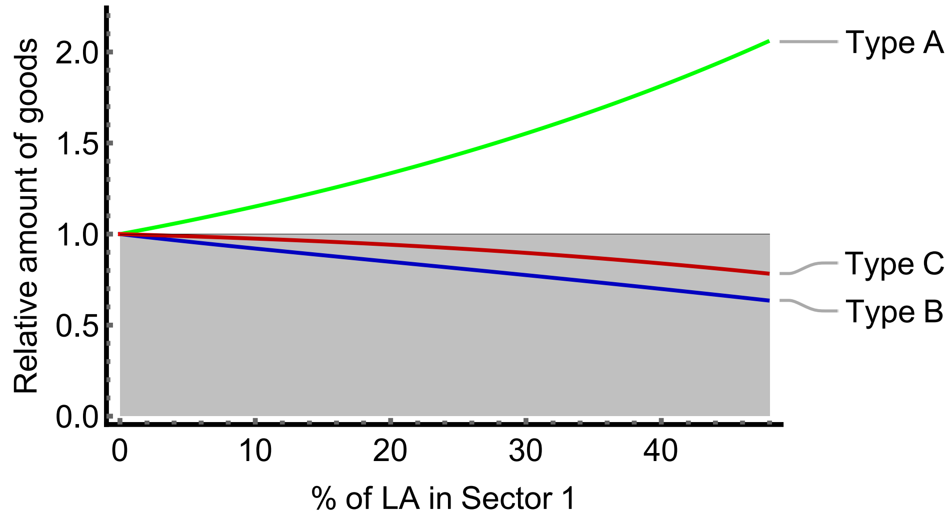

Fig. 12. A comparison of fraction of the pre-automation goods package that a worker can purchase after automation as compared with no automation. The shaded area is the region in which the worker is cannot buy the pre-automation bundle of goods (C.E.S., FO, Case 1).

Figure 12. alone does not prove that automation is good for A and bad for B and C workers because this single test does not address the question of who can pay for the post-automation package of goods under the pre-automation conditions. Figure 13 below shows the fraction of today's package of goods obtainable at pre-automation prices and income.

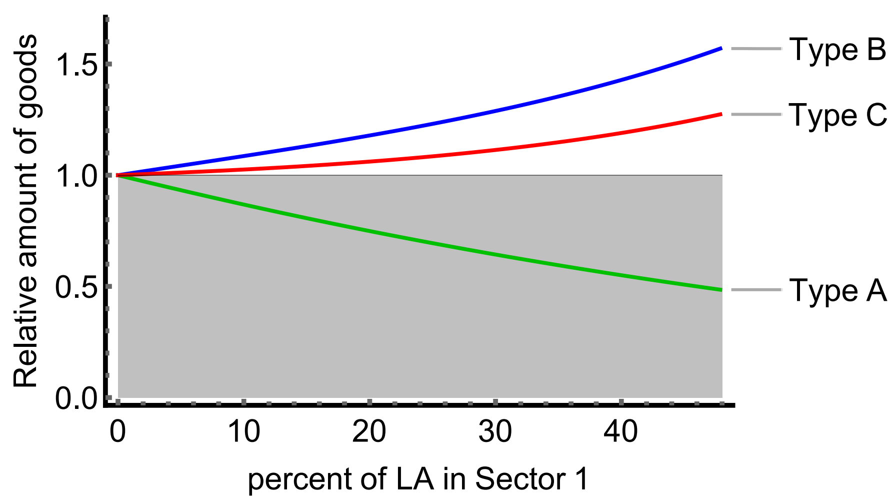

Fig. 13. A comparison of fraction of the post-automation goods package that a worker can purchase at pre-automation prices and income. The shaded area is the region in which the worker cannot buy as much at no automation prices as they can after automation (C.E.S., FO Case 1). If workers could buy the package they chose post-automation at pre-automation prices and income, they were better off pre-automation.

Taken together Figs. 12 and 13, demonstrate that automation is good for A and bad for B and C workers. They specifically prove that the decrease in goods prices arising from automation is not sufficient to offset the decline in the income of Type B and C workers.

4.4 Displacement

The loss of a job can be a traumatic experience for any worker. A displaced worker faces the cost associated with the search for a new job. Finding a satisfactory new job may involve moving to an area where jobs are more plentiful. Such a move can require the sale of their home in a depressed market and the need for the employed spouse as well as the displaced worker to find a new job. Even if a job can be found without moving, it can involve a harrowing period of search in a poor job market.

How large is the displacement workers by automation? Figures 14 and 15 illustrate the fraction of workers who will have to search for a new job after being displaced from their job by automation in Case 1.

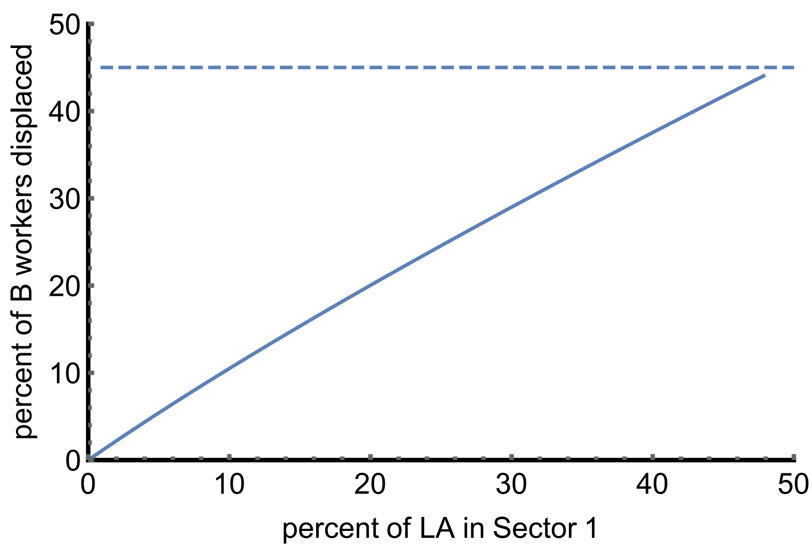

Fig. 14. The percentage of B workers who have experienced at least one displacement by automation plotted versus percent of L_A in Sector 1 (CES, Case1). The dashed line indicates the maximum percentage displacement of B workers predicted from Frey and Osborne's results.

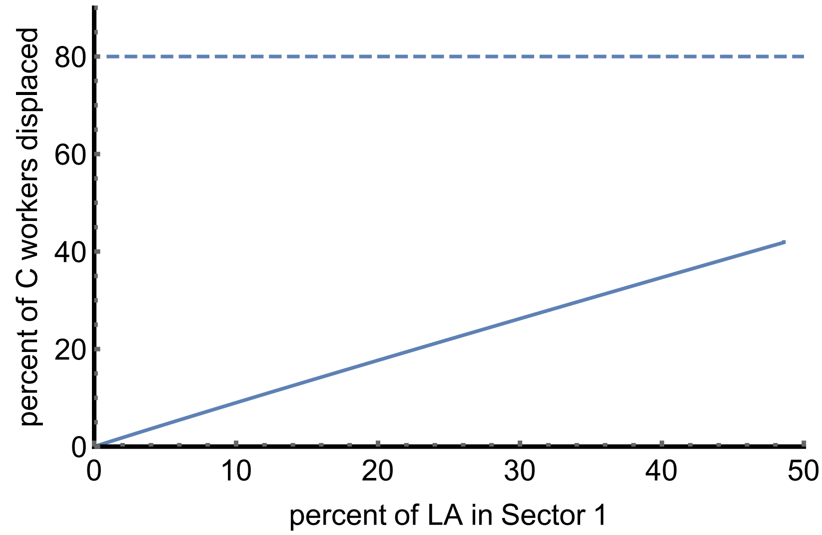

Fig. 15. The percentage of C workers who have experienced at least one displacement by automation plotted versus percent of L_A in Sector 1 (CES, Case1, FO). The dashed line indicates the maximum fractional displacement possible.

C workers do not approach its limit of 80% of workers displaced because they are not yet automated. At the extreme right of these figures $n_B = \bar{n}$ (=400000), but n_C is 190445 at this extreme.

As previously noted, automation is not exhausted when n_B equals $\bar{n}$ even though automation has been completed for L_B because automation has not been completed for L_C. L_B is automated more readily than L_C simply because B jobs pay better making them better targets for automation. Figure 16 illustrates what happens to the displacement of C workers.

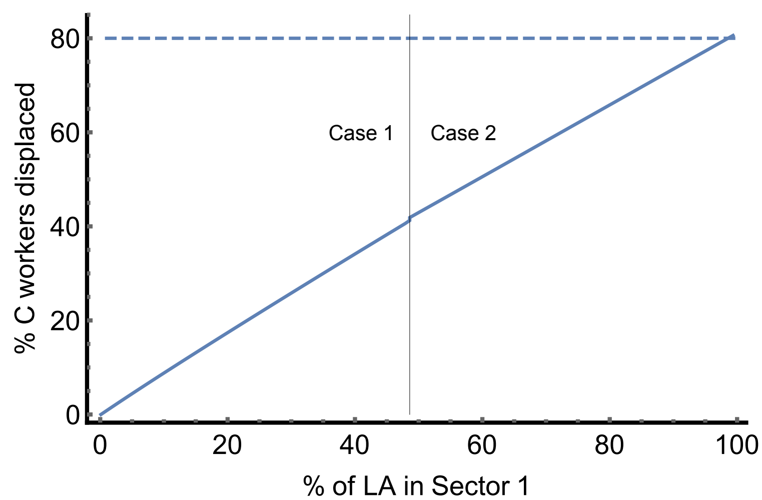

Fig. 16. Displacement of C workers to the completion of automation of C jobs. Instead it is the number of robots in the single sector that contains both kinds of robots (CES, FO).

Losing your job once and being forced into searching for another job, which is unlikely to pay as much can be emotionally traumatizing. Losing it again is even more traumatizing. Figure 17 plots the fraction of B workers who lose their jobs more than once.

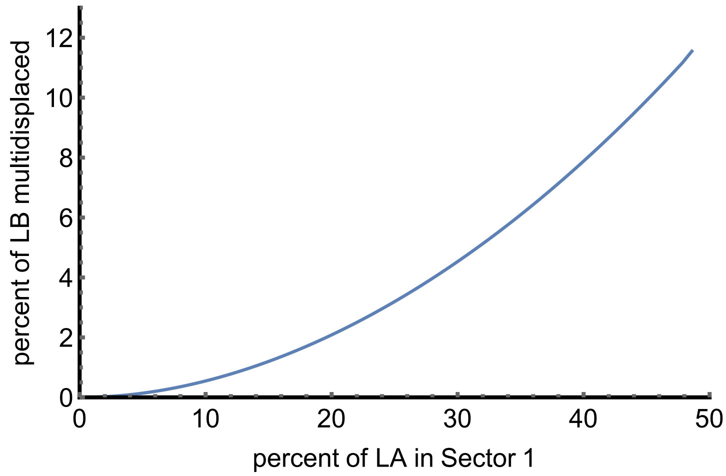

Fig. 17. The percentage of B workers predicted to be displaced more than once by automation. All B has been displaced by the end of Case 1 (CES, FO).

Figure 18 illustrates this multiple job loss situation for C workers.

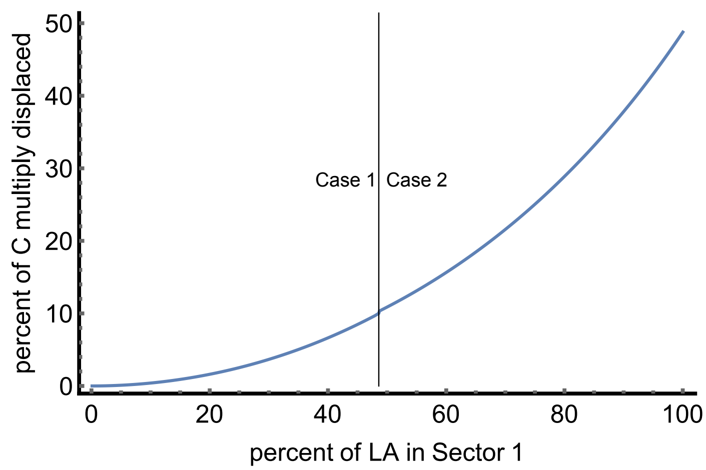

Fig. 18. Percentage of C workers predicted to experience more than one displacement by automation (CES, FO)

Even over the range of B automation (Case 1, FO), C workers are almost as likely to experience multiple job loss as B workers.

4.5 Effect of a minimum wage

Up to now, the effects of inflation have been mathematically irrelevant because everything so far considered involves in some sense only ratios. When wages are considered, inflation matters. The federal minimum wage has not kept up with inflation. As the reader may recall, our model economy is based upon 1980 data. Therefore, in order to maintain a selfconsistent model, we have decided that the best strategy is not to attempt to bring inflation into the model to make it more relevant to the present day. In 1980 the minimum wage was \$3.10 per hour or assuming 2000 hours of employment \$6200 per year. We will assume that value in calculating its effects and stick to the 1980 economy.

The \$6200 per year minimum wage affects Type A and B workers only indirectly as their wages are never as low as that. We choose to ignore any indirect effects on A and B focusing only how the minimum wage affects C employment.

In Figure 19, unemployment is plotted for an assumed minimum wage of \$6200 per year.

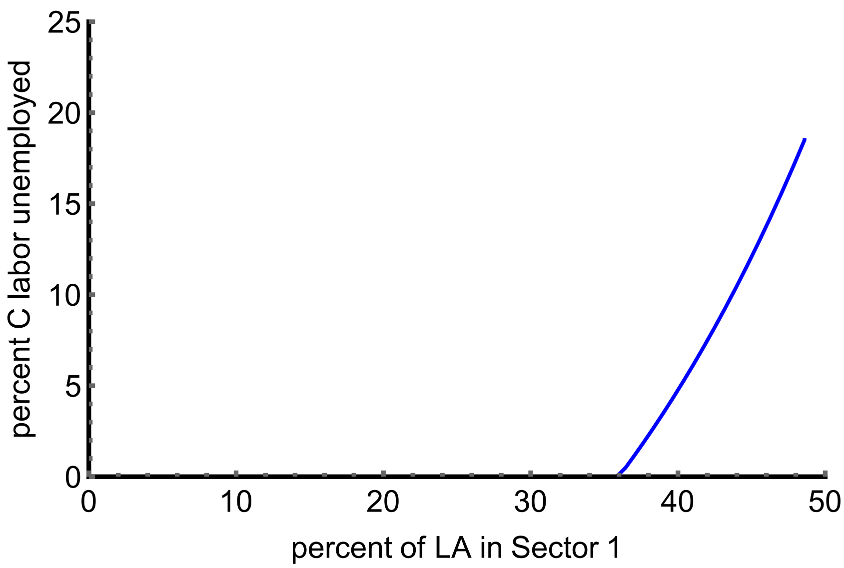

Fig. 19. The effect of automation upon unemployment using 1980 parameters combined with the 1980 minimum wage of \$3.10 per hour or \$6200 per year (Case 1, FO).

5. Conclusions

The wages of both B and C workers are set by the cost of automation at their respective margins. Decreasing the price of automation drives the wages of both B and C workers down. For the C.E.S. model in a competitive economy, we find that capital's share and the share of the high-skilled Type A workers increase at the expense of less-skilled workers. In 1980, the ratio of the income of the top ten percent divided by the income of the bottom ninety percent was slightly greater than one. We measure the extent of automation by the percentage of Type A workers who are in the sector containing both kinds of robots. With the Frey and Osborne C.E.S. parameters when this measure reaches 30%, the ratio of the income of the top ten percent divided by the income of the bottom ninety percent roughly doubles whether measured in wages or total income (i.e. wages plus returns on capital), and the labor GINI has increased from about 39 to about 50.[28] This market allocation of income may not be politically viable; presumably making it necessary to intervene with policies that redistribute income.

6. Discussion

6.1 Empirical evidence supporting our conclusions

Papers[29] have appeared suggesting that Frey and Osborne are wrong. The community of people who are interested in this subject appear to be focused on job loss by automation. Because automation has not manifested themselves as lost jobs,[30] the community belief seems to be that the effects of automation are small. In this they seem to have ignored the argument of McKinsey&Co that tasks not jobs are lost to automation. Our model assumes full employment[31] with its principal effect of automation fixing the wages of Type B and Type C workers to the marginal cost of robot subsitutes. Thus the effect of automation is not job loss, but lower wages for these workers as robots serve the function of the reserve army of the unemployed. In our models, GDP is growing while wages are falling. As shown in Figure 1, the U.S. economy is demonstrating that the median wage is not keeping up with the growth of the GDP. The cause for concern is not job loss, but increasing inequality.

6.1 Societal implications

The large increase in inequality of income made possible by automation will lead to an acceleration of the growth of the inequality of wealth. As Piketty has pointed out, the rate at which wealth accumulates is determined by income minus consumption. For the wealthy, additional income does not usually lead to a commensurate increase in consumption.[32] Our expectation is that automation will cause inequality in terms of wealth to grow enormously

[28] The *trend* is what matters not the absolute amount. GINI indices from various sources vary. FRED gives an overall GINI of 0.414 in 2015 for the U.S.

[29] e. g. L. Nedelkoska and G. Quinitini (2018) and M. Coelli and J. Borland (2019)

[30] Until COVID-19, unemployment in the U.S. economy was at record lows.

[31] Except for the single case with a minimum wage that we examine.

[32] An extreme example is Warren Buffet. Berkshire-Hathaway's last and only dividend was in 1967.

because middle income and lower income workers will consume almost all their income while the fraction of the income saved at the top will continue to increase.

6.2 Another look at our assumptions

The assumption that the capital is competitive at some fixed rate might be questioned. However, unlike workers, capital flows freely around the world seeking the best returns for acceptable risk. Trying to model capital would make the model very complicated, and we would need to make assumptions about savings and depreciation that would be open to question.

We primarily used the results of Frey and Osborne to calibrate our model, but the results of Brandes and Wattenhofer are not qualitatively different. Both predict that the jobs with greater potential for automation are usually low paying. Low wage jobs offer a potentially greater scope for automation, but in all of our models the progression of automation to replace lower wage jobs (C) is less rapid than that of higher wage jobs (B). This is caused by our choice of tying together the cost of automation. We could have had different drivers for automation for Type B and Type C workers, e.g. a β_B and β_C. However, we have no *a priori* knowledge of whether β_B is larger than β_C or smaller than β_C. Therefore, we chose to have only a single factor, $\beta = \beta_B = \beta_C$. This avoids introducing another parameter that we have no way to estimate. With that assumption, middle wage jobs pay higher wages and thus can be automated by more expensive robots.

We have made the work of Frey and Osborne[33] the foundation of our model. There have been several publications[34] that are deeply skeptical about Frey and Osborne's work. Brandes and Wattenhofer[35] in particular examined in detail every job that Frey and Osborne analyzed. We find that our calculations based on the work of Brandes and Wattenhofer soften the harsh effects of Frey and Osborne, but do not change our principal conclusions. We discuss objections on other grounds in Appendix C.

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[33] C. Frey and M. Osborne, 2017

[34] M. Coelli and F. Borland, 2019. L. Nedelkoska and G. Quintini, 2018 are examples.

[35] P. Brandes and R. Wattenhofer, 2017.

6.3 Commentary

Conceptually, one can imagine a model that addresses completely the effects of technical change upon labor, and introduces technical change that augments all classes of labor as well as introducing the creation of new jobs into a formal general equilibrium mathematical model. However, this would result in a model with many as yet unknown parameters. The thought of adding parameters made us recall John von Neumann's well-known[36] remark: "With four parameters I can fit an elephant and with five I can make him wiggle his trunk." Thus, we believe it necessary for us to make strong assumptions to restrict the number of free parameters.

Before automation had any great impact, management had already found other ways to reduce labor costs. Labor unions were an important factor in creating and maintaining well-paying middle skilled jobs in the past. When manufacturers moved to states with "right to work" laws, the wage premium attached to manufacturing began to be eliminated. This move of manufacturing south was followed by outsourcing to low wage countries. These changes have had an enormous impact upon wages in certain sectors. Will the lowered labor costs caused by automation bring manufacturing back to the U.S.? If so, will this eventually result in better wages for Type B and C workers? Our model would suggest "No" as the answer to the latter question.

Economists who believe the creation of new jobs driven by new industries will occur base this belief on the historical record that the technologies of the 20th century did not result in making labor redundant as new jobs were created. In order to support the notion of that new jobs for the middle class and lower class are going to improve their income, it is necessary to try to imagine what tasks can be performed by the displaced workers that cannot be performed by robots. Undoubtedly there will be some, but their accompanying jobs will not be high paying unless they involve abilities that are scarce in the population of displaced workers. If not, the quasi-rents will be eliminated as more workers acquire the necessary skills to compete for those jobs. To us, it not about how many jobs there are, but

[36] Attributed to von Neumann by Enrico Fermi, as quoted by Freeman Dyson in "A meeting with Enrico Fermi" in *Nature* 427 (22 January 2004) p. 297

instead about the marginal product of workers being fixed to the marginal product of robots.

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Appendices

Introduction

In order to observe the effect of different choices of elasticity of our production function, we have carried out the calculation with a Cobb-Douglas production function as well as a CES production function with an elasticity of 0.5. We believe the CES choice is more realistic, but we have found no analytical solution for the CES production function. There are analytical, albeit complicated, solutions for the Cobb-Douglas production function. We describe this in Appendix A so that the interested can develop an understanding of how the problem is structured. The CES with elasticity 1/2 production functions requires the numerical solution of three (or, with added complexity, two) more difficult factor constraints. The approaches that can be used to determine solutions to the constraints problem for CES. will be described in Appendix B. Appendix C discusses criticisms leveled at Frey and Osborne's work.

Appendix A: Analysis of the problem with a Cobb-Douglas production function

Section A.I Structure of the problem

To structure the demand market, we assume that there is a representative consumer with a utility function[37]

(A.I.1)
$$W = \int_0^{\bar{n}} \alpha \, ln\left[y(n)\right] dn$$

The consequence (A.I.1) is that the amount spent on each good is equal to the amount spent on every other good. We will define the value of α by defining the good produced by the $\bar{n}$ firm as the numeraire good. This defines the price of all goods produced without robots.

The resource constraints are:

[37] This is a common assumption in the literature. See for example Acemoglu and Restrepo 2017, 2018, or Arntz, Gregory, and Zierahn 2016.

(A.I.2) $$\overline{L}_A = \int_0^{\overline{n}} l_A(n)dn$$

(A.I.3) $$\overline{L}_B = \int_0^{\overline{n}} \left[l_{B1}(n) + l_{B2}(n) \right] dn$$

(A.I.4) $$\overline{L}_C = \int_0^{\overline{n}} \left[l_{C1}(n) + l_{C2}(n) \right] dn$$

$\overline{L}_A$, $\overline{L}_B$ and $\overline{L}_C$ are the amounts of Type A, Type B and Type C workers available. The subscripts represent current job assignments.

If we assume a Cobb-Douglas production function, an *n*-firm has production functions of the form

(A.I.5) $$y(n) = Ak^a(n) l_A^b(n) \left[l_{B1}(n) + r_B(n) \right]^{c_1} l_{B2}^{c_2} \left[l_{C1}(n) + r_C(n) \right]^{d_1} l_{C2}^{d_2}$$

We are maximizing an integral subject to integral constraints and some of the control variables have corners. Thus, it is necessary to use Pontryagin's Maximum Theorem to solve the problem.

The constraints (A.I.2), (A.I.3) and (A.I.4) can be converted into differential equations with the appropriate endpoints and λ_3, λ_4, and λ_5 are the co-state variables in Pontryagin's Theorem.

The Hamiltonian for the planner's problem is

(A.I.6)

$$\begin{aligned} H = {} & \alpha \ln\left[y(n) \right] - \lambda_1 \left\{ Ak^a(n) l_A^b(n) \left[l_{B1}(n) + r_B(n) \right]^{c_1} l_{B2}^{c_2} \left[l_{C1}(n) + r_C(n) \right]^{d_1} l_{C2}^{d_2} - y(n) \right\} \\ & - \hat{\lambda}_2 \left[k(n) + \left(\alpha 0_B \beta + n\beta \right) r_B(n) + \left(\alpha 0_C \beta + n\beta \right) r_C(n) \right] - \lambda_3 l_A(n) \\ & - \lambda_4 \left(l_{B1}(n) + l_{B2}(n) \right) - \lambda_5 \left(l_{C1}(n) + l_{C2}(n) \right) \end{aligned}$$

where $\hat{\lambda}_2$ is the price of $k(n)$ capital, $\hat{\lambda}_2(\alpha 0_B\beta+n\beta)$ is the price of $r_B(n)$ robots and $\hat{\lambda}_2(\alpha 0_C\beta+n\beta)$ is the price of $r_C(n)$ robots where $\alpha 0_B$ and $\alpha 0_C$ are chosen such that $r_B(0)=0$ and $r_C(0)=0$ when β=1. The driving variable for this problem is not time as is normally the case, but *n*. The next to last term in the Hamiltonian forces B1 workers and B2 workers to get the same wage and the last term in *H* forces C1 workers and C2 workers to get the same wage.

We are using the results of Frey and Osborne to determine the value of the parameters of the economy. For these values, automation progresses more rapidly for B workers than for C workers (A workers are exempted for reasons discussed in the text) with the result automation is complete for B workers before it is complete for C workers. The problem breaks down into two Cases. In Case 1, automation of B workers is incomplete; in Case 2 automation of B workers has been completed. All B workers remain employed because some tasks in some jobs cannot be automated. There is yet a third case, which we do not choose to treat here, after Case 2 in which automation of both B and C workers is complete.

Section A.II Division of the economy into Sectors

The Hamiltonian is maximized using the Kuhn-Tucker Theorem. The solution is defined in 3 open intervals $(0,n_C),(n_C,n_B)\,and\,(n_B,\bar{n})$ where $\bar{n}$ is last firm in the economy. The Kuhn-Tucker conditions are

From the Cobb-Douglas property of constant shares, for all β and *n*.

(A.II.1) $$\lambda_1(n)y(n)=\alpha$$

(A.II.2) $$k(n)=\lambda_1(n)a\frac{y(n)}{\hat{\lambda}_2}=a\frac{\alpha}{\hat{\lambda}_2}$$

(A.II.3) $$l_A(n)=\lambda_1(n)b\frac{y(n)}{\lambda_3}=b\frac{\alpha}{\lambda_3}$$

(A.II.4) $$l_{B2}(n) = \lambda_1(n) c_2 \frac{y(n)}{\lambda_4}$$

(A.II.5) $$l_{C2}(n) = \lambda_1(n) d_2 \frac{y(n)}{\lambda_5} = d_2 \frac{\alpha}{\lambda_5}$$

The equations A.II.1 to A.II.5 apply in all three intervals.

For the interval $(0, n_C)$ Sector 1

(A.II.6) $$r_C(n) = \lambda_1(n) d_1 \frac{y(n)}{\lambda_2(\alpha 0_C + n)\beta}$$

The corner point n_C

The Kuhn-Tucker conditions with respect to $l_{C1}(n)$, *and* $r_C(n)$ involve the possibility of a corner. Define n_C such that for $n > n_C$. (Note the strict inequality, $n > n_C$, and that $r(n_C) = 0.$)

(A.II.8) $$l_{C1}(n)\left[\lambda_1(n) d_1 \frac{y(n)}{l_{C1}(n)} - \lambda_5\right] = 0$$

and for $n < n_C$ (again, note the strict inequality, $n < n_C$, and that $l_{C1}(n) = 0.$)

(A.II.9) $$r_C(n)\left[\lambda_1(n) d_1 \frac{y(n)}{r_C(n)} - \hat{\lambda}_2(\alpha 0_C + n)\beta\right] = 0$$

Now consider the point where $n = n_C$. At that point $l^*_{C1}(n_C) > 0$ and/or $r^*(n_C) > 0$ so that the term $\left[l_{C1}(n_C) + r_C(n_C)\right] > 0$. A necessary condition for both A.II.8 and A.II.9 to be simultaneously true is

(A.II.10) $$\lambda_5 = \hat{\lambda}_2(\alpha 0_C \beta + n_C \beta)$$

(A.11.10) is the connection between the cost of a robot at the corner point and the market wage of Type C workers.

The corner point n_B

Define n_B such that for $n > n_B$. (Note the strict inequality, $n > n_B$, and that $r\left(n_B\right) = 0.$)

(A.II.11)
$$l_{B1}\left(n\right)\left[\lambda_1\left(n\right)c_1\frac{y(n)}{l_{B1}\left(n\right)} - \lambda_4\right] = 0$$

and $n < n_B$ (again, note the strict inequality, $n < n_B$, and that $l_{B1}\left(n\right) = 0.$)

(A.II.12)
$$r_B\left(n\right)\left[\lambda_1\left(n\right)c_1\frac{y(n)}{r_B\left(n\right)} - \hat{\lambda}_2\left(\alpha 0_B + n\right)\beta\right] = 0$$

Now consider the point where $n = n_B$. At that point $l^*_{B1}\left(n_B\right) > 0$ and/or $r^*\left(n_B\right) > 0$ so that the term $\left[l_{B1}\left(n_B\right) + r_B\left(n_B\right)\right] > 0$. Thus

(A.II.13)
$$\lambda_4 = \hat{\lambda}_2\left(\alpha 0_B + n_B\right)\beta$$

A.II.13 is the connection between the cost of B robots at the corner with the market wage of Type B workers.

For the interval $\left(n_C, n_B\right)$ Sector 2

$n > n_C$, $n < n_B$, $r_C\left(n\right) = 0$, $r_B\left(n\right) > 0$

Restating

(A.II.12a)
$$r_B\left(n\right) = \lambda_1\left(n\right)c_1\frac{y(n)}{\left(\alpha 0_B\beta + n\beta\right)}$$

Restating

((A.II.6a) $$l_{C1}(n) = \lambda_1(n) d_1 \frac{y(n)}{\lambda_5} = d_1 \frac{\alpha}{\lambda_5}$$

The equation A.II.6a also applies in Sector 3 below.

Section A.III Sector 3 (no robots here)
We turn our attention to calculating the production for a single good in the all human sector (Sector 3). From the first order conditions we know

(A.III.1) $l_A^*(n)$ is independent of *n* and β.

(A.III.2) $l_{B2}^*(n)$ is independent of n.

(A.III.3) $l_{C2}^*(n)$ is independent of n.

Because l_{B1} and l_{B2} are in the same market, they are tied together

(A.III.4) $$l_{B2}^* = \frac{c_2}{c_1} l_{B1}^*$$

Making use of the fact that the l_{B1} displaced in Sector 1 and Sector 2 is now in Sector 3 ($n \geq n_B$)

(A.III.5) $$\frac{c_2}{c_1} l_{B1}^* \bar{n} + l_{B1}^*(\bar{n} - n_B) = \bar{L}_B$$

Solving A.III.5 for l_{B1}^* and n_B we have

(A.III.6) $$l_{B1}^* = \frac{c_1 \bar{L}_B}{(c_1 + c_2)\bar{n} - n_B c_1}$$

rearranging we have

(A.III.7) $$n_B = \frac{(c_1 + c_2)\bar{n} l_{B1}^* - c_1 \bar{L}_B}{c_1 l_{B1}^*}$$

By an exactly analogous process (which we leave to the reader) using the corner condition a n_C and the constraint of $\overline{L}_C$ we determine l^*_{C1} and n_C to be

(A.III.8)
$$l^*_{C1} = \frac{d_1 \overline{L}_C}{(d_1 + d_2)\overline{n} - d_1 n_C}$$

(A.III.9)
$$n_C = \frac{(d_1 + d_2)\overline{n} l^*_{B1} - d_1 \overline{L}_C}{d_1 l^*_{C1}}$$

Section A.IV Developing a solution for $\alpha \equiv y_H$

The goods in Sector 3 have identical production functions. We choose to make the price of good produced by the $\overline{n} - th$ firm the numeraire good thereby making $\alpha = y_H$ for any good in Sector 3. Choosing a generic Sector 3 good, its production can be written.

(A.IV.1)
$$\alpha \equiv y = Ak^a l_A^b l_{B1}^{c_1} l_{B2}^{c_2} l_{C1}^{d_1} l_{C2}^{d_2}$$

where we have suppressed identifying *y* as belonging to the all human sector 3 and any dependence on *n* because all goods in this sector have identical production functions.

We regard β as the quantity we have introduced artificially to mimic the growth of automation technology making n_B and α as quantities to be calculated. However, the most convenient way forward to an analytical solution of the problem will be to temporarily regard n_B as an independent variable and eliminate the role of β in solving for α.

In most constraint optimization where the inputs are constrained the solution is independent of output, i.e. α. However, in this problem capital is not constant making the solution depend upon α. We constrain the solution of the problem by requiring that that α be the value of the $\overline{n} - th$ firm making that good the numeraire good. Thus, the problem can be defined as following: for any give β find the value of n_B such that $\lambda_1(\overline{n}) y(\overline{n}) = \alpha$ if $\lambda_1(\overline{n}) = 1.$

To begin, $l_A = \overline{L}_A / \overline{n}$ and A.III.5 gives an expression for l^*_{B1} in terms on n_B. Now, our motivation to regard n_B as the independent variable emerges. We can solve A.III.5 for l^*_{B1}

without invoking β and insert the result into A.IV.1 and use the relation of A.III.4 to obtain l^*_{B2}.

The corner conditions allow us to express α in two ways

(A.IV.2) $$\alpha = \frac{\lambda_4^* l_{B1}^*}{c_1} = \frac{\lambda_5^* l_{C1}^*}{d_1}$$

When A.III.6 is substituted for l^*_{B1}, A.II.13 is substituted for λ_4^*, and A.II.10 is substituted for λ_5^*, we obtain

(A.IV.3) $$\frac{\hat{\lambda}_2 \beta(\alpha 0_B + n_B)}{c_1} \frac{c_1 \bar{L}_B}{(c_1+c_2)\bar{n} - n_B c_1} = \frac{\hat{\lambda}_2 \beta(\alpha 0_C + n_C) l^*_{C1}}{d_1}$$

We can cancel out $\hat{\lambda}_2$ and β in (A.IV.3), but we still have an equation with n_C in it. We can use (A.III.9) to eliminate it and get

(A.IV.5) $$\frac{\hat{\lambda}_2 \beta(\alpha 0_B + n_B)}{c_1} \frac{\bar{L}_B}{\bar{n}} = \frac{\hat{\lambda}_2 \beta}{d_1} \left[\alpha 0_C + \frac{(d_1+d_2)\bar{n} l^*_{B1} - d_1 \bar{L}_C}{d_2 l^*_{C1}} \right] l^*_{C1}$$

that can be solved for l^*_{C1} .The resulting expression is reproduced below (β has canceled out in the process).

(A.IV.6) $$l^*_{C1} = \frac{d_1 \left[\bar{n}(c_1+c_2)\bar{L}_C + d_1 \bar{L}_B (n_B + \alpha 0_B) - c_1 \bar{L}_C n_B \right]}{\left[(c_1+c_2)\bar{n} - c_1 n_B \right] \left[d_2 \bar{n} + d_1 (\bar{n} + \alpha 0_C) \right]}$$

Returning to (A.IV.1), we know now the values of $l^*_{B1}, l^*_{B2}, l^*_{C1}$ *and* l^*_{C2} .

This leaves *k* as the final factor in A.IV.1 to be addressed. There is no factor constraint on *k*. Its value is fixed by market for capital at a fixed cost of $\hat{\lambda}_2$ and its value is fixed by

(A.IV.7) $$k = \frac{ay}{\hat{\lambda}_2}$$

We can substitute (A.IV.7) into (A.IV.1) giving

(A.IV.8)
$$\alpha = \left(\frac{a}{\hat{\lambda}_2} \right)^{\frac{a}{(1-a)}} \left(A l_A^b l_{B1}^{c_1} l_{B2}^{c_2} l_{C1}^{d_1} l_{C2}^{d_2} \right)^{\frac{1}{(1-a)}}$$

substituting all the known quantities into (A.IV.8) yields.

(A.IV.9)
$$\alpha(n_B) = \left(\frac{a}{\hat{\lambda}_2} \right)^{\frac{a}{(1-a)}} \left(A \left(\frac{L_A}{\overline{n}} \right)^b \left(\frac{c_2}{c_1} \right)^{c_2} \left[\frac{c_1 \overline{L}_B}{(c_1+c_2)\overline{n} - n_B c_1} \right]^{c_1+c_2} \left(\frac{d_2}{d_1} \right)^{d_2} \left\{ \frac{d_1 \left[\overline{n}(c_1+c_2)\overline{L}_C + d_1 \overline{L}_B (n_B + \alpha 0_B) - c_1 \overline{L}_C n_B \right]}{\left[(c_1+c_2)\overline{n} - c_1 n_B \right] \left[d_2 \overline{n} + d_1 (\overline{n} + \alpha 0_C) \right]} \right\}^{d_1+d_2} \right)^{\frac{1}{(1-a)}}$$

Section A.V Methods of solution

Because A.IV.9 does not contain β, there is an almost philosophical choice to be made in the structure of results here. By making n_B the primary variable, one can obtain α as a function of n_B and then knowing α calculate β by solving (A.III.8) for β. In calculating a table of the values of α, it will be equally spaced in n_B and unequally spaced in β.

Now the corner condition A.II.13 can be used

(A.V.1)
$$\lambda_4^* = c_1 \frac{\alpha}{l_{B1}^*(n_B)} = \hat{\lambda}_2 (\alpha 0_B + n_B) \beta$$

with A.III.6 to create an expression for β in terms of α and n_B.

(A.V.2)
$$\beta = \frac{[(c_1+c_2)\overline{n} - c_1 n_B]\alpha}{\hat{\lambda}_2 \overline{L}_B (\alpha 0_B + n_B)}$$

Alternatively, you can express α in terms of β and n_B by solving A.V.2 for α obtaining

(A.V.3)
$$\alpha = \frac{\overline{L}_B (\alpha 0_B + n_B) \beta \hat{\lambda}_2}{(c_1+c_2)\overline{n} - c_1 n_B}$$

following up to determine n_B by numerically finding the root of the equation

(A.V.4) $$\frac{\bar{L}_B(\alpha 0_B + n_B)\beta\hat{\lambda}_2}{(c_1+c_2)\bar{n} - c_1 n_B} - \alpha(n_B) = 0$$

for n_B in terms of β.

Section A.VI.1 Case 2

After $n_B = \bar{n}$ and $n_C < \bar{n}$, the all human Sector 3 no longer exists. There are no firms that do not use robots. The $\bar{n}$ firm that produces the numeraire good now uses r_B robots to instead of l^*_{B1} labor. The appropriate production function for the Sector 2 firm is

(A.VI.1) $$y(n) = Ak(n)^a l_A(n)^b r_B(n)^{c_1} l_{B2}(n)^{c_2} l_{C1}(n)^{d_1} l_{C2}(n)^{d_2}$$

where r^*_B has replaced l^*_{B1}. There are no longer any r^*_B robots competing directly with B workers, but r^*_C robots still compete with C workers at n_C and A.III.9 still applies. Now the plan is to calculate

(A.VI.2) $$\alpha \equiv y(\bar{n}) = Ak(\bar{n})^a l_A(\bar{n})^b r_B(\bar{n})^{c_1} l_{B2}(\bar{n})^{c_2} l_{C1}(\bar{n})^{d_1} l_{C2}(\bar{n})^{d_2}$$

All factors except l_{C1} and l_{C2} are decoupled with the new aspect being the presence of $r_B(\bar{n})$. The value of *k* for any *n* is still

(A.IV.6) $$k = \frac{ay}{\hat{\lambda}_2}$$

and *k* can be replaced in $y(\bar{n})$ by the same method as described in Case 1.

(A.VI.3) $$\alpha(\bar{n}) = \frac{\hat{\lambda}_2(\alpha 0_B + \bar{n})\beta r_B(\bar{n})}{c_1} = \frac{\hat{\lambda}_2(\alpha 0_C + n_C)\beta l^*_{C1}}{d_1}$$

The factors limiting α are the L_A and L_C constraints as determined in

(A.III.9) $$l_{C1}^{*} = \frac{d_1 \overline{L}_C}{(d_1 + d_2)\overline{n} - d_1 n_C}$$

The value of $r_B^{*}(\overline{n})$ is determined by

(A.VI.4) $$\frac{\hat{\lambda}_2(\alpha 0_B + \overline{n})\beta r_B^{*}(\overline{n})}{c_1} = \frac{\hat{\lambda}_2(\alpha 0_C + n_C)\beta \overline{L}_C}{(d_1 + d_2)\ \overline{n} - d_1 n_C}$$

that can be solved to give

(A.VI.5) $$r_B^{*}(\overline{n}) = \frac{\left[c_1 \overline{L}_C\ (n_C + \alpha 0_C)\right]}{\left[(d_1 + d_2)\overline{n} - d_1 n_C)\right](\overline{n} + \alpha 0_B)}$$

l_{B2} is now given by

(A.VI.6) $$l_{B2}^{*} = \frac{\overline{L}_B}{\overline{n}}$$

and $l_A^{*} = \frac{\overline{L}_A}{\overline{n}}$ as before. After all the substitutions described in this section are made in $y(\overline{n})$, the expression for $y(\overline{n})$ is only a function of n_C and is written as

(A.VI.7) $$y(\overline{n}) = \left\{ \frac{a^a A c_1^{c_1} d_1^{d_1} d_2^{d_2} \overline{L}_A^{b} \overline{L}_B^{c_2} \overline{L}_C^{c_1+d_1+d_2} (\alpha 0_C + n_C)^{c_1}}{\overline{n}^{b+c_2} \left[(d_1 + d_2)\overline{n} - d_1 n_C\right]^{c_1+d_1+d_2} \hat{\lambda}_2^{a} (\alpha 0_B + \overline{n})^{c_1}} \right\}^{\frac{1}{1-a}}$$

Because (A.VI.7) does not contain β, $y(\overline{n})$ can be determined as a function n_C and the resulting α used with n_C to determine β or by finding the root of (A.VI.8) below by proceeding as in Case 1 using n_C instead of n_B replacing (A.V.4) with

(A.VI.8) $$\frac{\overline{L}_C \hat{\lambda}_2 (\alpha 0_C + n_C)\beta}{(d_1 + d_2)\overline{n} - d_1 n_C} - y(n_C) = 0$$

to determine first n_C for any particular value of β that limits n_C to the range determined by the upper limit of Case 1 and $\bar{n}$. The calculation of output and factor shares is straightforward using the properties of Cobb-Douglas.

Appendix B: C.E.S. calculational overview

Case 1

As in the Cobb-Douglas economy, there are two corners, n_C and n_B, that must be found. Once again, any good produced by humans only can be made the numeraire, thereby defining α. In the robot containing sectors, y[n] is no longer constant unlike the Cobb-Douglas economy. We have not been able to find any analytical solution for elasticity one-half C.E.S. The locations of both corners have to be found simultaneously by numerical calculation.

The most straightforward approach to optimizing production subject to factor constraints is to start with assumed values of L_{AH}, L_{BH}, L_{CH}. Since the cost of capital, $q \equiv \hat{\lambda}_2$, is known, these three quantities are sufficient to determine all properties in the all human sector in particular K_H, w_A, w_B, and w_C. Once the four wages, (q, w_A, w_B, w_C) are known, the two sectors containing robots can solved to give the factors in those sectors. Thus, the problem becomes determining the values of L_{AH}, L_{BH}, L_{CH} consistent with the factor constraints, L_A, L_B, L_C.

Because L_{AH}, L_{BH}, L_{CH} are known when β=1, the search can be carried out starting there by decreasing β slightly. The L_A, L_B, L_C constraints are expanded in the three independent variables (L_{AH}, L_{BH}, L_{CH}) in a Taylor series using numerical differentiation keeping only the constant and linear terms. The resulting set of three linear equations is solved repeatedly to obtain successively better approximations to values of L_{AH}, L_{BH}, L_{CH} that satisfy the constraints. This approach works quite well for large values of β. However, as β gets smaller, L_{AH}, L_{BH}, L_{CH} interact in an increasingly nonlinear manner requiring ever smaller steps in decreasing β. Nevertheless, this approach works.

There are two hidden relations that taken together create a relation between the L_A factor constraint and the L_B factor constraint. Using this relation reduces the parameter space to be searched from three dimensional to two dimensional leading to a search of two factor constraints in the space of n_B and n_C.

The first method requires searching in a space with three unknowns while the second requires searching in a space with only two unknowns. In the first method, α, n_B, and n_C must all be determined. The two hidden relations permit the human sector to be completely determined if only n_B, and n_C are known. Since the value of production, α, is based on the production of any human good, there is no need to use the third constraint which combines the constraint on B with the constraint on A. Thus, the first method requires the use of all three (A, B, and C labor) factor constraints while the second invokes only the A and C labor factor constraints.[38]

Case 2

In Case 2, $n_B = \bar{n}$. This eliminates the sector containing only human workers. Making the good at $\bar{n}$ the numeraire, there is a smooth connection of Case 2 to Case 1 particularly in the determination of α..

The two factor constraints become

(B1) $$L_{AR1} + L_{AR2} = L_A$$

(B2) $$L_{CR1} + L_{CR2} = L_C$$

We use these factor constraints to solve for w_A and n_C. The functions that we have already developed for L_{AR1}, L_{AR2}, L_{CR1}, L_{CR2} are already in terms of w_A and n_C along with other quantities that can readily be determined from w_A and n_C. Once again, we start with the known β at which $n_B = \bar{n}$ and decrease β in small steps using the Taylor expansion of the dependence of Eqs. B1 and B2 upon w_A and n_C. Once w_A and n_C are determined, quantities such as factor shares and inequality ratios are readily calculated for Case 2.[39]

Appendix C: Criticisms of Frey and Osborne

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[38] The constraint on L_B could be used in combination with the constraint on L_C instead.

[39] The *Mathematica* Notebooks used are available in the Supplemental Material.

A number of criticisms have been aimed at Frey and Osborne's paper say that they have proposed a much larger number of jobs lost to automation than it is realistic to expect. These criticisms offer two separate and completely orthogonal evidences. A very few (one group, only Brandes and Wettenofer[40] as far as we can find) have reevaluated[41] the choices that Frey and Osborne made in a way that we can apply. We have used the results (predictions-work_activities.csv) from their web-site to recalculate our most important results. We have not repeated every aspect our calculations using Brandes and Wattenhofer's result. We have not been able to tell which of these two inputs is the closer to reality.

Criticisms based upon no apparent job loss

The other criticisms,[42] which are made more often, is that economic changes have not manifested themselves as lost jobs. In this, they seem to have ignored the argument of McKinsey&Co. that tasks, not jobs are lost to automation. As far as the U.S. economy is concerned, we argue that the evidence is showing strong effects of displacements by automation. Our model assumes full employment, but workers can be displaced so that the principal effect of automation is to fix wages of Type B and Type C workers to the marginal cost of automation. Thus the effect of automation is not job loss, but lowered real wages, real GDP growth and rising inequality.

[40] P. Brandes and R. Wattenhofer (2016)

[41] M. Coelli and J. Borland (2019) seem to say that to have done this, but do not provide the data we need perhaps because they do not accept the methodology of Frey and Osborne.

[42] e. g. L. Nedelkoska and G. Quinitini (2018) and M. Coelli and J Borland (2019)